Global Networks, Monetary Policy and Trade*

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Abstract

We develop a multi-country, multi-sector New Keynesian model with incomplete financial markets, input–output linkages, and heterogeneous sectoral price rigidities to study the macroeconomic effects of tariffs. Tariffs act simultaneously as demand and supply shocks. A risk-sharing wedge—terms-of-trade effects and revalued net foreign assets—summarizes the wealth transfer in general equilibrium and determines whether the tariff-imposing country gains or loses. This wedge interacts with a country-sector-time propagation matrix encoding network position, sectoral rigidity, and cross-country monetary policy heterogeneity to shape consumption and exchange rate dynamics. Through input–output linkages, transitory tariffs generate persistent real marginal cost distortions, unlike New Keynesian benchmarks with one-time price jumps. These distortions exceed what N-country monetary policy can offset, even under flexible exchange rates. Quantitatively, the 2025–2026 tariffs are stagflationary for the U.S. and yield inflation or deflation abroad depending on trade diversion and monetary heterogeneity. Tariff threats alone generate inflation in the U.S. and in other countries, depending on retaliation expectations.

JEL Codes: E2, E3, E6, F1, F4

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Online Appendix available here.

1 Introduction

This paper studies the macroeconomic impact of trade distortions, theoretically and quantitatively, using a novel open-economy New Keynesian framework (NKOE). Understanding the aggregate consequences of trade distortions requires confronting three features that canonical models treat in isolation: global input-output (I-O) linkages, sectorally heterogeneous nominal rigidities, and incomplete financial markets. We build a multi-country, multi-sector NKOE model incorporating these features and ask: what is the macroeconomic impact of tariffs, and how does a global production network shape that impact?

Tariffs transmit to the macroeconomy through three channels. First, tariffs act as demand shocks. By raising the domestic price of foreign goods, they shift expenditure from imports toward domestically produced varieties. As relative demand shocks, they reallocate spending from taxed to untaxed varieties rather than changing aggregate demand. With finite elasticity of substitution across varieties, taxing a subset of goods also impacts the aggregate consumption-basket price.

Second, tariffs act as supply shocks. Modern production networks are global and complex: firms rely on imported intermediate inputs that are complementary to each other, and taxes on those inputs flow directly into marginal costs, impacting producer prices.

Third, tariffs generate wealth transfers. Under incomplete markets, households cannot fully insure against shocks and tariff-induced relative price changes redistribute resources across countries in global general equilibrium. We summarize these wealth transfers through a risk-sharing wedge, the deviation of relative consumption across countries from the complete-markets Backus–Smith benchmark. In response to a tariff shock, the wedge opens as a one-time martingale, and hence the long-run effect is priced-in on impact reflecting the permanent change in wealth. The sign and magnitude of the wedge help determine consumption dynamics and are shaped by two opposing forces. The wedge is negative when favorable terms-of-trade movements and balance-sheet gains on net foreign assets produce a one-time wealth transfer toward home large enough to dominate intertemporal substitution. Then, even though tariffs make today a more expensive period to consume, the wealth gain raises home consumption on impact. The wedge is positive when terms of trade move against home and net foreign asset positions deteriorate; both forces push wealth abroad and home consumption down.

Whether the home country is a net winner or loser depends on three primitives: the global production network structure, which governs the strength of the supply shock and the terms-of-trade response; elasticities of substitution across goods, which govern how easily one variety substitutes for another and hence interaction between demand and the supply shock; and the persistence of the tariff shock, which governs the strength of intertemporal substitution relative to the wealth effect impacting the time path of consumption.

Our N-country, J-sector model, featuring Rotemberg pricing, portfolio adjustment costs, an empirically relevant mix of producer- and dominant-currency pricing, and a Taylor rule, features all these channels. The linearized equilibrium is characterized by five vector equations: an IS curve, a Phillips curve for producer prices, a CPI definition, an Uncovered Interest Parity (UIP) condition, and a balance-of-payments (BoP) equation, nesting a broad class of NKOE models. We solve the model analytically in two blocks, an NK block and a BoP block, by extending the method of undetermined coefficients to matrix scale.

The production network matters for the macroeconomic impact of tariffs for two reasons. First, it impacts terms of trade and the sign of the risk-sharing wedge. Without intermediate inputs, the wedge is negative for standard parameterizations, so tariffs favor the tariff-imposing country. Once intermediate inputs are included, home dependence on foreign inputs can move terms of trade against home. To see this, suppose home imports cars and also semiconductors used to produce chips, and imposes tariffs on both. This hurts home chip production while raising demand for home cars. Foreign car production is hurt, since foreign firms need home chips, making them relatively more scarce and valuable in spite of the higher demand for home cars, tilting terms of trade against home, in favor of foreign.

Second, the production network generates persistence in real marginal cost deviations, governed by the NKOE propagation matrix $\mathbf{\Psi}^{\text{NKOE}}$. This matrix is the coefficient on lagged real marginal costs and determines how past cost distortions feed into current inflation and output. The result requires more than one sector: with $J = 1$, there is one sectoral price distortion per country and $\mathbf{\Psi}^{\text{NKOE}} = \mathbf{0}$, while with $J > 1$ there are NJ sectoral distortions and $\mathbf{\Psi}^{\text{NKOE}} \neq \mathbf{0}$. Conditional on $J > 1$, stronger intermediate-input linkages (e.g., a higher share of imported inputs/foreign input dependence) make inherited cost distortions unwind more slowly and thus raise inflation persistence. Propagation depends on the number of sectors, sectoral price rigidity, and cross-country monetary policy, while the persistence threshold remains $J > 1$ regardless of whether the exchange rate is treated as a separate national instrument. Country-level aggregate demand cannot span all sectoral distortions and hence N-dimensional policy tools cannot offset NJ inertia-inducing lagged states.

We apply the model to the 2025–2026 U.S. tariff episode.[1] We conduct two exercises. First, we feed the country–sector tariffs implemented in 2025–26 using applied border rates

[1] We first validate the model against the 2018 U.S.–China trade war, for which it predicts a 3–4% nominal dollar appreciation against the yuan, a 0.1pp decline in U.S. real GDP, and a 0.27pp increase in U.S. inflation. These are broadly consistent with available estimates: in 2018, the dollar appreciated by about 5.6%, inflation rose by 0.1–0.2pp, and aggregate real income fell by about 0.04% of GDP (Barbiero and Stein, 2025; Fajgelbaum et al., 2020).

in each quarter. In the medium run, these tariffs are stagflationary for the U.S. and generate sizable international spillovers, including trade diversion from the U.S. toward Europe. This process is persistent up to 10 quarters (if policy looks-through), leading to a cumulative rise of 0.37pp in the U.S. inflation. Counterfactuals show that open-economy models without I-O linkages can overstate the inflationary impact and understate the output decline, missing slow-moving propagation across countries and time. Second, we study reversed tariff threats, where the home country announces future tariffs that are withdrawn before implementation. Even a credible threat that is never implemented generates sizable macro effects through the expectations channel alone: U.S. inflation rises 0.34pp on impact and does not return to steady state for some time, despite the reversal being announced after a single period.

Our positive analysis carries normative implications. The persistence result implies that tariffs in a multi-sector economy generate distortions that take longer to unwind. We quantify this by comparing sticky- and flexible-price impulse responses to the same tariff shock: with I-O linkages and $J > 1$, the sticky-price allocation takes several additional quarters to converge to the flexible-price allocation. Whether the resulting stabilization burden falls on inflation, output, or both depends on the policy rule.

The paper proceeds as follows. Section 2 presents the baseline model. Section 3 analyzes the flexible-price solution and characterizes the risk-sharing wedge. Section 4 introduces nominal rigidities, derives the NKOE propagation matrix, and establishes why production networks generate persistence in real marginal cost deviations. Section 5 presents the quantitative analysis and policy counterfactuals. Section 6 concludes.

1.1 Relation and contribution to literature.

Our paper combines and builds on three distinct literatures: NKOE, networks and trade.

We extend the NKOE literature's canonical two-country models of Obstfeld and Rogoff (1995) and Clarida et al. (2002) to N countries and J sectors with endogenous current accounts, showing the importance of I–O linkages and N-country monetary policies for global imbalances. A large part of the NKOE literature focuses on SOEs, such as Barattieri et al. (2021), omitting intermediate input imports.[2] Recent work adds intermediate inputs but not full I-O linkages (e.g., Auray et al., 2024; Ambrosino et al., 2024; Auclert et al., 2025).[3] A related SOE literature studies optimal monetary policy under tariff shocks such as Bergin and Corsetti (2023), Bianchi and Coulibaly (2025), Werning et al. (2025), and Monacelli (2025).

[2] This literature, rooted in Galí and Monacelli (2005), focuses on optimal exchange-rate and monetary policies.

[3] A few recent papers also incorporate cross-border production networks or multilateral trade linkages, including Qiu et al. (2025), Cuba-Borda et al. (2025), and Ho et al. (2022). Relative to these papers, our contribution is to deliver a closed-form analytical characterization of tariff transmission and persistence.

Our contribution to this literature is showing how the presence of production networks can change benchmark results from the one-sector SOE baseline in global general equilibrium by impacting inflation-output trade-off and persistence of inflation.

The closed-economy NK-networks literature establishes that productivity shocks induce endogenous cost-push effects and welfare losses, that the Phillips curve is flatter with production networks (Rubbo, 2023; Pasten et al., 2020, 2024; Afrouzi et al., 2024), and that when sectoral prices outnumber aggregate policy instruments, monetary policy cannot close all sectoral gaps simultaneously (Guerrieri et al., 2021; La'O and Tahbaz-Salehi, 2022). We show analytically that the *persistence of real marginal cost deviations is larger with global I-O linkages.* This is in the spirit of Afrouzi and Bhattarai (2023), which studies monetary and sectoral shocks in a continuous-time closed-economy NK production network. We differ by studying tariffs in a discrete-time open economy with global I-O linkages and incomplete risk sharing. Tariffs generate a cross-country wealth transfer, summarized by the martingale risk-sharing wedge, that shifts the inflation-output tradeoff. While the exchange rate is an additional choice variable, it does not change the $J > 1$ threshold for persistence. Differing from closed-economy propagation, $\mathbf{\Psi}^{\text{NKOE}}$ contains the impact of the exchange rate: heterogeneous monetary policy maps into exchange-rate movements, and imported-input linkages feed them back into sectoral marginal costs. Monetary policy heterogeneity can increase persistence relative to the closed-economy benchmark.

We build on the quantitative general-equilibrium trade models (e.g., Caliendo and Parro, 2015; Baqaee and Farhi, 2024) by adding dynamics, nominal rigidities, and incomplete markets. Our model's long-run flexible-price equilibrium nests a structure similar to these frameworks. The contribution of our framework is the short-to-medium-run transitional dynamics: nominal rigidities generate persistent deviations from the long-run allocation, and the speed of convergence depends on the network structure and monetary policy. We share the importance of incomplete markets with Itskhoki and Mukhin (2025), who study the long-run impact of tariffs on trade balances and optimal tariff policies.[4] Our results also carry a methodological implication for tariff analysis. Static trade models with exogenous transfers may overstate the tariff-induced wealth transfer, while complete-markets models force it to the foreign country by construction. Since this wedge shapes the consumption response and enters the inflation–output trade-off, it is central to whether tariffs are expansionary on impact. A tractable incomplete-markets structure is thus an essential ingredient of tariff analysis.

[4]In the long run, Costinot and Werning (2025) show that trade deficits depends on the extensive margin. We have a similar result in our long-run flexible-price equilibrium.

Our risk-sharing wedge is conceptually related to the Backus–Smith wedge in Aguiar et al. (2025), who show how exchange rate disconnect can open this wedge.

2 Environment

We develop a multi-country, multi-sector New Keynesian model that incorporates nominal rigidities via Rotemberg costs and portfolio adjustment costs.

2.1 Households

The household in country n maximizes the present value of lifetime utility:

$$\max_{\{C_{n,t},L_{n,t},B^{US}_{n,t},B_{n,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{n,t}^{1-\sigma}}{1-\sigma} - \chi \frac{L_{n,t}^{1+\eta}}{1+\eta} \right]$$

subject to:

$$P^C_{n,t} C_{n,t} + T_{n,t} - B_{n,t} - \mathcal{E}^{US}_{n,t} B^{US}_{n,t} + \mathcal{E}^{US}_{n,t} P^{US}_t \psi(B^{US}_{n,t}/P^{US}_t) =$$
$$W_{n,t} L_{n,t} + \sum_i \mathcal{D}_{ni,t} - (1+i_{n,t-1}) B_{n,t-1} - \mathcal{E}^{US}_{n,t}(1+i^{US}_{t-1}) B^{US}_{n,t-1},$$

where $P^C_{n,t}$ is the price of the consumption bundle ($C_{n,t}$) at time t, $\mathcal{E}^{US}_{n,t}$ is the exchange rate between country n and the U.S. (an increase denotes local-currency depreciation), $W_{n,t}$ is the wage, $L_{n,t}$ is labor supply, $i_{n,t}$ is the nominal interest rate on the local-currency bond $B_{n,t}$, and i^{US}_t is the interest rate on the U.S. bond $B^{US}_{n,t}$, which are net foreign liabilities. The term $\psi(B^{US}_{n,t}/P^{US}_t)$ is a stationarity-inducing portfolio adjustment cost ensuring a unique steady-state level of real debt. $T_{n,t}$ denotes taxes and transfers, including lump-sum rebates of tariff revenue, and $\mathcal{D}_{ni,t}$ denotes firm profits. β is the discount factor, σ the intertemporal elasticity of substitution, χ the labor disutility weight, and η the labor supply elasticity. The domestic bond is in net zero supply; all countries save or dissave using U.S. bonds. Beyond the UIP condition, arbitrage ensures consistency of bilateral exchange rates with each country's rate against the U.S. In our model, tariffs are revenue-neutral; tariff revenue is rebated back to domestic households in a lump-sum manner through the $T_{n,t}$ term.

Maximizing the household's lifetime utility subject to the budget constraints yields:

Euler Equation: $1 = \beta E_t \left[\left(\frac{C_{n,t+1}}{C_{n,t}} \right)^{-\sigma} \frac{P^C_{n,t}}{P^C_{n,t+1}} (1+i_{n,t}) \right] \forall n \in N$ and $\forall t$ and

UIP Condition: $\frac{1+i_{n,t}}{1+i^{US}_t} = E_t \left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}} \right] \frac{1}{1-\psi'(B^{US}_{n,t}/P^{US}_t)}$ $\forall n \in N-1$ (excluding the U.S).

Finally, for completeness of notation, we define arbitrage conditions: $\mathcal{E}_{n,m,t} = \mathcal{E}^{US}_{n,t}/\mathcal{E}^{US}_{m,t}$ $\forall n, m \in N$, which also pins down a country's exchange rate with itself (with $\mathcal{E}_{n,n,t} = 1$). We have $N \times N$ exchange rates, and along with the UIP condition, these two conditions uniquely determine the exchange rate.

We now turn to the household's intratemporal problem. The first part of the intratemporal problem is the standard labor-consumption trade-off that determines labor supply $W_{n,t}/P^C_{n,t} = \chi L^{\eta}_{n,t} C^{\sigma}_{n,t}$ where $W_{n,t}$ is the wage in country n at time t.

Determining the intratemporal breakdown of consumption involves a nested CES structure. Outputs from different countries are first bundled into a country-sector consumption bundle, which is then aggregated into the country consumption bundle. For example, aggregate US consumption is a CES of all goods consumed including automobiles; and US consumption of automobiles is a CES aggregate of US consumption of automobiles from each country. Mathematically:

$$C_{n,t} = \left[\sum_{i \in J} \Gamma_{n,i}^{\frac{1}{\theta^C_h}} C_{n,i,t}^{\frac{\theta^C_h - 1}{\theta^C_h}} \right]^{\frac{\theta^C_h}{\theta^C_h - 1}} \quad \text{and} \quad C_{n,i,t} = \left[\sum_{m \in N} \Gamma_{n,i,mi}^{\frac{1}{\theta^C_{l,i}}} C_{n,i,mi,t}^{\frac{\theta^C_{l,i} - 1}{\theta^C_{l,i}}} \right]^{\frac{\theta^C_{l,i}}{\theta^C_{l,i} - 1}}.$$

Here, the index (n, i) captures the sector level (i) bundles in country n. $C_{n,i,t}$ is country n's consumption of sector bundle i and $\Gamma_{n,i}$ is the weight of the bundle i. θ^C_h is the elasticity that governs the substitution between different sectors in consumption (e.g., between automobiles and food in consumption). This bundle is then a combination of all goods of i procured by country n from countries $m \in N$ globally. The sectoral bundle (i) in country n is formed by country-sector varieties (mi), which we index with (n, i, mi). $\Gamma_{n,i,mi}$ is the weight of country m's good in this bundle (e.g., German automobiles –mi– in automobile bundle –i– for the U.S. consumers – n). $\theta^C_{l,i}$ is the elasticity of substitution between different country varieties in sector i. Prices and consumption levels of this object are indexed the same way. We can then express the optimality conditions in line with the CES structure:

$$P^C_{n,t} = \left[\sum_{i \in J} \Gamma_{n,i} (P^C_{n,i,t})^{1-\theta^C_h} \right]^{\frac{1}{1-\theta^C_h}} \quad \text{and} \quad C_{n,i,t} = \Gamma_{n,i} \left(\frac{P^C_{n,i,t}}{P^C_{n,t}} \right)^{-\theta^C_h} C_{n,t},$$

where $P^C_{n,i,t}$ is the local currency consumption price of the aggregated good basket i in country n at time t.[5]

The bundle $C_{n,i,t}$ aggregates varieties sourced from different countries. Let $P_{mi,t}$ denote the output price of industry i in country m, expressed in m's currency. To express this price in country n's currency, we apply the bilateral exchange rate $\mathcal{E}_{n,m,t}$. Additionally, country n levies a tariff $\tau_{n,mi,t}$ on country-sector mi. The price of good mi faced by buyers in country n is therefore $P_{n,mi,t} = \mathcal{E}_{n,m,t}(1 + \tau_{n,mi,t})P_{mi,t}$, and the associated price index of the bundle $C_{n,i,t}$ is given by:

[5] We use the superscript C to denote price bundles in the consumption side.

$$P^C_{n,i,t} = \left[\sum_{m\in N} \Gamma_{n,i,mi} P_{n,mi,t}^{1-\theta^C_{l,i}}\right]^{\frac{1}{1-\theta^C_{l,i}}} \quad \text{and} \quad C_{n,mi,t} = \Gamma_{n,i,mi} \left(\frac{P_{n,mi,t}}{P^C_{n,i,t}}\right)^{-\theta^C_{l,i}} C_{n,i,t}.$$

2.2 Production

Having defined the household's side, we now turn to the production side of the economy. Output in country n, sector i, at time t follows a CES production function:

$$Y_{ni,t} = \left[\alpha_{ni}^{1/\theta^X} L_{ni,t}^{\frac{\theta^X-1}{\theta^X}} + (1-\alpha_{ni})^{1/\theta^X} (X_{ni,t})^{\frac{\theta^X-1}{\theta^X}}\right]^{\frac{\theta^X}{\theta^X-1}} \forall n \in N, \forall i \in J,$$

where $Y_{ni,t}$ is the output of sector i in country n, θ^X governs the elasticity between the labor and intermediate bundle $X_{ni,t}$, and α_{ni} is the labor weight. All firms within a given country-sector combination are assumed to be identical, and each firm solves the following marginal cost minimization problem:

$$MC_{ni,t} = \min_{\{X_{ni,j,t}, L_{ni,t}\}} W_{n,t} L_{ni,t} + P^X_{ni,t} X_{ni,t} \quad \text{s.t.} \quad Y_{ni,t} = 1,$$

where $P^X_{ni,t}$ is the price of the intermediate bundle for country-sector ni (we use the superscript X to denote price indices in the production side, where all price indices are given by the relevant CES dual).

A firm solving this problem chooses labor and the quantities of industry-specific intermediate goods. The intermediate bundle for ni is built from sectoral bundles indexed by (ni, j, t) for all $j \in \mathcal{J}$, where each sectoral bundle aggregates sector-j varieties sourced from different countries. For instance, producing automobiles (i) in the U.S. (n) requires steel (j) potentially sourced from any $m \in \mathcal{M}$. The corresponding bundle and relative demand condition are

$$X_{ni,j,t} = \left[\sum_{m\in N} \Omega_{ni,j,mj}^{\frac{1}{\theta^X_{l,j}}} X_{ni,mj,t}^{\frac{\theta^X_{l,j}-1}{\theta^X_{l,j}}}\right]^{\frac{\theta^X_{l,j}}{\theta^X_{l,j}-1}} \text{and } X_{ni,mj,t} = \Omega_{ni,j,mj} \left(\frac{P_{n,mj,t}}{P^X_{ni,j,t}}\right)^{-\theta^X_{l,j}} X_{ni,j,t},$$

where $P^X_{ni,j,t}$ is the price index for this bundle, and $X_{ni,j,t}$ is the quantity. $\theta^X_{l,j}$ governs the elasticity of substitution among different varieties within sector j on the production side. The aggregate intermediate bundle for ni is given by:

$$X_{ni,t} = \left[\sum_{j\in J} \Omega_{ni,j}^{\frac{1}{\theta^X_h}} X_{ni,j,t}^{\frac{\theta^X_h-1}{\theta^X_h}}\right]^{\frac{\theta^X_h}{\theta^X_h-1}} \text{and } \frac{X_{ni,j,t}}{X_{ni,t}} = \Omega_{ni,j} \left(\frac{P^X_{ni,j,t}}{P^X_{ni,t}}\right)^{-\theta^X_h}, \forall j \in \mathcal{J}.$$

Given the setup and definitions above, the firm's problem additionally yields the following equilibrium conditions:

$$MC_{ni,t} = \left[\alpha_{ni} W_{n,t}^{1-\theta^X} + (1-\alpha_{ni}) \left(P_{ni,t}^X\right)^{1-\theta^X}\right]^{\frac{1}{1-\theta^X}} \quad \text{and} \quad \frac{X_{ni,t}}{L_{ni,t}} = \frac{(1-\alpha_{ni})}{\alpha_{ni}} \left(\frac{W_{n,t}}{P_{ni,t}^X}\right)^{\theta^X}.$$

Within each country sector there is an infinite continuum of identical firms. Representative firm f in country-sector ni sets its price subject to Rotemberg adjustment costs:

$$P_{ni,t}^f = \arg\max_{P_{ni,t}^f} \mathbb{E}_t \Bigg[\sum_{T=t}^{\infty} \text{SDF}_{t,T} \Bigg[Y_{ni,T}^f(P_{ni,T}^f) \left(P_{ni,T}^f - MC_{ni,T}\right)$$
$$- \tfrac{(1-\vartheta_{ni})\delta_{ni}}{2} \left(\frac{P_{ni,T}^f}{P_{ni,T-1}^f} - 1 \right)^2 Y_{ni,T} P_{ni,T} - \tfrac{\vartheta_{ni}\delta_{ni}}{2} \left(\frac{P_{ni,T}^f / \mathcal{E}_{n,T}^{US}}{P_{ni,T-1}^f / \mathcal{E}_{n,T-1}^{US}} - 1 \right)^2 Y_{ni,T} P_{ni,T} \Bigg]\Bigg]$$

where a CES bundler puts together the output of individual firms such that the demand function is $Y_{ni,t}^f(P_{ni,t}^f) = \left(P_{ni,t}^f / P_{ni,t}\right)^{-\theta^R} Y_{ni,t}$. In this Rotemberg setup, $(1-\vartheta_{ni})\delta_{ni}$ captures the real cost of changing the price in producer currency and $\vartheta_{ni}\delta_{ni}$ captures the real cost of changing the price in the dominant currency (US dollar), with ϑ_{ni} denoting the share of prices that are rigid in the dominant currency rather than the producer currency. Thus, $\vartheta_{ni} \to 0$ corresponds to PCP, $\vartheta_{ni} \to 1$ corresponds to DCP, and $\vartheta_{ni} \in (0,1)$ yields a hybrid of the two pricing schemes. In our quantitative work in Section 5 we additionally incorporate pricing to destination market (PTM) by relabeling sectors in a destination-specific manner, so that a given sector i is relabeled as the sector that produces i for destination market m. This introduces producer-importer specific producer prices that are sticky (e.g., the producer price of Japanese steel *intended for American end users* is sticky). We discipline ϑ_{ni} using the share of exports invoiced in dollars established by the DCP literature and assume this share is zero for goods intended for the domestic market. The resulting framework combines PCP, DCP, and PTM in a manner consistent with empirical evidence.

This problem yields the following equilibrium condition:

$$(1-\vartheta_{ni})(\Pi_{ni,t}-1)\Pi_{ni,t} + \vartheta_{ni} \left(\frac{\Pi_{ni,t}}{D_{n,t}^{US}} - 1 \right) \frac{\Pi_{ni,t}}{D_{n,t}^{US}} = \frac{\theta^R}{\delta_{ni}} \left[\frac{MC_{ni,t}}{P_{ni,t}} - \frac{\theta^R - 1}{\theta^R} \right]$$
$$+ \beta\, \mathbb{E}_t \left[(1-\vartheta_{ni})(\Pi_{ni,t+1}-1)\Pi_{ni,t+1} + \vartheta_{ni} \left(\frac{\Pi_{ni,t+1}}{D_{n,t+1}^{US}} - 1 \right) \frac{\Pi_{ni,t+1}}{D_{n,t+1}^{US}} \right] \quad (1)$$

where $\Pi_{ni,t}$ is gross inflation ($\Pi_{ni,t} = P_{ni,t}/P_{ni,t-1}$) and $D_{n,t}^{US}$ is gross depreciation of the producer's currency against the USD ($D_{n,t}^{US} = \mathcal{E}_{n,t}^{US}/\mathcal{E}_{n,t-1}^{US}$). Equation (1) constitutes a country- and sector-specific forward-looking New Keynesian Phillips Curve, expressed in terms of nominal marginal cost deflated by the sector's producer price. As $\delta_{ni} \to 0$, the flexible-price allocation with a constant markup is recovered: $\Pi_{ni,t} = 1$ and $MC_{ni,t}/P_{ni,t} = (\theta^R - 1)/\theta^R$.

Following convention, our linearization and quantitative analysis are conducted around a subsidized steady state that removes the steady-state monopolistic distortion. We omit the explicit notation for this subsidy to conserve notation and do not consider exogenous markup shocks. Tariff shocks nevertheless induce endogenous markup variation through nominal rigidities, so we track real marginal costs.[6]

This issue brings up the question of whether pre-tariff and pre-exchange rate producer prices or prices after tariffs and exchange rate adjustment are sticky. Our framework is flexible to accommodate both. In the baseline specification, under both PCP ($\vartheta_{ni} = 0$) and DCP ($\vartheta_{ni} = 1$) tariffs are applied on top of the sticky producer price, so they pass through immediately. Since full instant pass-through for end-users from tariffs is not empirically realistic, in Section 5 we introduce domestic importing firms, whose job is to distribute and conduct retail sales of imports; all imports go through this sector. This makes passthrough of tariffs to consumers more realistic and serves to restore the distribution margin absent from standard I-O tables (Horowitz and Planting, 2009).

2.3 Definitions, Market Clearing, and Policy Equilibrium

We track the evolution of each country's net debt with the balance of payments equation. For country n, we can write the balance of payments equation as (with tariff revenue rebated back to households):

$$\begin{aligned}
&\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}}(P_{n,mj,t}C_{n,mj,t}) + \sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}}(P_{n,mj,t}X_{ni,mj,t}) + \mathcal{E}_{n,t}(1+i^{US}_{t-1})B^{US}_{n,t-1} \\
&+ \mathcal{E}_{n,t}P^{US}_t\psi(B^{US}_{n,t}/P^{US}_t) = \sum_{i\in\mathcal{J}} P_{ni,t}Y_{ni,t} + \sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}}\left(\frac{\tau_{n,mj,t}}{1+\tau_{n,mj,t}}P_{n,mj,t}C_{n,mj,t}\right) \\
&+ \sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}}\left(\frac{\tau_{n,mj,t}}{1+\tau_{n,mj,t}}P_{n,mj,t}X_{ni,mj,t}\right) + \mathcal{E}_{n,t}B^{US}_{n,t} \quad \forall n \in N-1.
\end{aligned} \tag{2}$$

All terms in this equation are written in terms of the domestic currency of country n. The second and third terms on the right hand side correspond to the tariff revenues. Given market-clearing conditions and budget constraints, one country's budget constraint is redundant as an equilibrium condition; thus, we omit that of the first country, which corresponds to the U.S. in our model.

All markets clear. The market-clearing condition for labor is ($L_{n,t} = \sum_{i\in J} L_{ni,t}$). The market for USD bonds clears: $B^{US}_t = \sum_{n=2}^{N} B^{US}_{n,t}$, where $B^{US}_t = B^{US}_{1,t}$ is the aggregate bond supply of the US. Goods can be used as final (consumption) goods and as intermediate

[6] Markups also move because wages and intermediate-input prices need not adjust at the same rate.

inputs in all countries. Therefore, we write the goods market-clearing condition for country-sector ni at time t as follows: $Y_{ni,t} = \sum_{n\in\mathcal{N}} C_{m,ni,t} + \sum_{m\in\mathcal{N}} \sum_{j\in\mathcal{J}} X_{mj,ni,t}$, where country m is the consuming country and n is the producing country.

To close the model, we specify policy. Monetary policy in each country follows a generalized rule allowing for interest rate smoothing and targeting of a generic price basket:

$$P^T_{n,t} = \left(\prod_{m\in N}\prod_{j\in J}\left(P_{mj,t}\right)^{\Upsilon^P_{n,mj}}\right)\left(\prod_{m\in N\setminus\{n\}}\mathcal{E}_{n,m,t}^{\Upsilon^{\mathcal{E}}_{n,m}}\right) \quad \text{and} \quad \Pi^T_{n,t} = \frac{P^T_{n,t}}{P^T_{n,t-1}}.$$

Here $\Upsilon^P_{n,mj}$ is the weight of the producer price $P_{mj,t}$ (country m, sector j) in country n's target basket, and $\Upsilon^{\mathcal{E}}_{n,m}$ is the weight on the bilateral exchange rate $\mathcal{E}_{n,m,t}$. This nests CPI targeting, combinations of producer prices, and exchange-rate targeting.[7] The Taylor rule with inertia is $1 + i_{n,t} = (1 + i_{n,t-1})^{\rho^n_m}\left(\Pi^T_{n,t}\right)^{\phi^n_\pi}$ for all $n \in N$, where $\rho^n_m \in [0, 1)$ governs interest-rate smoothing.

2.4 Linearization, Matrix Notation and Analytical Solution

We linearize the 25 equations above and define an approximated equilibrium in order to use the method of undetermined coefficients (MUC) and solve the model analytically. We use hat variables to denote percent deviation from the zero-tariff pre-shock steady state rather than the flexible-price allocation.[8] While open economy models are commonly linearized around a zero-debt steady state, we take a different approach and allow for asymmetry of the primitive parameters (i.e., home bias and imported intermediate input dependence) across countries (e.g., Obstfeld and Rogoff, 1995). This implies a certain level of debt and net exports at the steady state that has to be consistent with these parameters. This level of steady-state debt is then used to parametrize the portfolio adjustment costs that discourage deviations from steady-state levels of debt. In the quantitative section, we discipline the asymmetry of parameters and the steady-state level of debt using the ICIO Table (for details see Appendix B).

To solve the model analytically, we make simplifying assumptions. The first simplifying assumption involves adopting elastic labor in the spirit of Golosov and Lucas (2007) preferences. That is, we set $\chi = 1$ and $\eta = 0$, making labor infinitely elastic, which simplifies the intratemporal labor-leisure choice to $\hat{W}_{n,t} - \hat{P}_{n,t} = \sigma\hat{C}_{n,t}$. This simplification allows us to focus on consumption in our five-equation Global New Keynesian Representation and track aggregate output separately. The second simplifying assumption is to abstract from

[7]It also nests PPI targeting and the divine-coincidence index of Rubbo (2023), which places more weight on stickier sectors.

[8]Accordingly, our output variable is not an output gap. We compare post-shock output to pre-tariff steady-state output, as our interest is in the world with tariffs relative to a world without.

Table 1. Notation guide

	Scalar notation	Matrix/vector notation	Object
Key Primitives	γ_H, γ_F	$\boldsymbol{\Gamma}$ $(N \times NJ)$	consumption shares
	Ω_H, Ω_F	$\boldsymbol{\Omega}$ $(NJ \times NJ)$	input–output shares / production network
	θ^C, θ^X	$\boldsymbol{\theta}$	CES elasticity of substitution
	Λ_{ni}	$\boldsymbol{\Lambda}(NJ \times NJ)$	nominal rigidity parameters
	ϕ_π	$\boldsymbol{\Phi}$ $(N \times N)$	monetary-policy coefficients
Key Variables	$\hat{C}_{n,t}$	$\hat{\boldsymbol{C}}_t$ $(N \times 1)$	real consumption
	$\hat{P}_{ni,t}$	$\hat{\boldsymbol{P}}_t^P$ $(NJ \times 1)$	producer prices
	$\pi_{ni,t}^P$	$\boldsymbol{\pi}_t^P$ $(NJ \times 1)$	producer-price inflation
	$\hat{P}_{n,t}^C$	$\hat{\boldsymbol{P}}_t^C$ $(N \times 1)$	consumer prices
	$\mu_{ni,t}$	$\boldsymbol{\mu}_t$ $(NJ \times 1)$	real marginal cost
	$\hat{\mathcal{E}}_{n,m,t}$	$\hat{\boldsymbol{\mathcal{E}}}_t$ $(N^2 \times 1)$	nominal exchange rates
	$\hat{V}_{n,t}$	$\hat{\boldsymbol{V}}_t$ $(N \times 1)$	net external debt
	$\hat{\tau}_{n,mj,t}$	$\hat{\boldsymbol{\tau}}_t$ $(N^2 J \times 1)$	tariffs
Key Objects	$\hat{w}_t$	–	risk-sharing wedge
	–	$\boldsymbol{\Psi}$ $(NJ \times NJ)$	NKOE propagation matrix

Note: n, m index countries, i, j index industries, and boldface denotes stacked country or country-sector objects. Hat notation denotes deviation from steady state. Elasticity of substitution refers to CES bundles on the consumption and production side.

portfolio adjustment costs; that is $\psi(B_{n,t}^{US}/P_t^{US}) = 0$. Portfolio adjustment costs serve as a stationarity-inducing device in the model. Because, these costs are numerically small, in analytical work we simplify them away. Third, for analytical simplicity we assume price rigidity in the producer prices, i.e., $\vartheta_{ni} = 0\ \forall n \in N, i \in J$.

Additionally, we assume that policy targets only a basket of producer prices and not the exchange rate, setting $\Upsilon_{n,m}^{\mathcal{E}} = 0\ \forall n, m \in N$. In our analytical work, we use the weights that producer prices have in the consumption basket for the target basket such that $\hat{\boldsymbol{\imath}}_t = \boldsymbol{\Phi}\boldsymbol{\Gamma}\boldsymbol{\pi}_t^P$.[9] Finally, we introduce generalized elasticities that directly link the lowest-level bundles to the highest-level aggregates respectively on the consumption side and production side.[10]

Given the N-country, J-industry structure of the model, notation is necessarily dense and we utilize the matrix form. Table 1 collects the main primitives, key variables, and key objects that recur throughout the paper. To build up this notation, let us consider the

[9] Our results do not hinge on using the $\boldsymbol{\Gamma}$ matrix; one could instead target a different mix of producer prices.

[10] To the first order, bundles presented in Sections 2.1 and 2.2 can be directly linked to the goods that form them. In particular, we can write $\Gamma_{n,mi} = \Gamma_{n,i}\ \Gamma_{n,i,mi}$ and $\Omega_{ni,mj} = (1-\alpha_{ni})\ \Omega_{ni,j}\ \Omega_{ni,j,mj}$. With these weights, we can express highest-level aggregates in terms of the lowest-level bundles; for instance, $\hat{C}_{n,t} = \sum_{m \in N}\sum_{i \in J}\Gamma_{n,mi}\hat{C}_{n,mi,t}$ and $\hat{C}_{n,mi,t} = -\theta_{l,i}^C\left(\hat{P}_{mi,t} + \hat{\mathcal{E}}_{n,m,t} + \tau_{n,mi,t} - \hat{P}_{ni,t}\right)$.

linearized producer price inflation equation:

$$\pi^P_{ni,t} = \underbrace{\frac{\theta^R_{l,i}}{\delta_{ni}}}_{\Lambda_{ni}} \Bigg(\underbrace{\alpha_{ni}\hat{W}_{n,t} + \sum_{m\in N}\sum_{j\in J}\Omega_{ni,mj}(\hat{P}_{mj,t} + \hat{\mathcal{E}}_{n,m,t} + \hat{\tau}_{n,mj,t})}_{\widehat{MC}_{ni,t}} - \hat{P}_{ni,t} \Bigg) + \beta \mathbb{E}_t \pi^P_{ni,t+1} \tag{3}$$

This can be vectorized as $\boldsymbol{\pi}^P_t = \boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\hat{\boldsymbol{W}}_t + (\boldsymbol{\Omega} - \boldsymbol{I})\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^P_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}^P_{\tau}\hat{\boldsymbol{\tau}}_t\right) + \beta\mathbb{E}_t\boldsymbol{\pi}^P_{t+1}$ where with some slight abuse of notation, we define the $\hat{\boldsymbol{\mathcal{E}}}_t$ as the $N^2 \times 1$ vector of bilateral exchange rates, the $\hat{\boldsymbol{\tau}}_t$ as the $N^2 J \times 1$ vector of tariff rates. In line with these vector representations, we also use $\mathbf{L}$ to denote loadings (i.e., how the subscript variable loads onto the superscript variable as a linear combination of the entries of the vector variable).[11] These expressions compactly describe how vector variables load onto a given equation and serve as partial derivatives.

2.4.1 Global New Keynesian Representation

With vector and matrix notation established, the linearized equilibrium conditions in Appendix A can be written in vector form as an equilibrium satisfying the Blanchard-Kahn stability conditions, which we use both for interpretation and to solve the model via MUC.[12] This five-equation representation extends the canonical three-equation New Keynesian model to N open economies with I-O linkages.

Definition 1. A linearized equilibrium comprises sequences $\{\hat{\boldsymbol{C}}_t, \hat{\boldsymbol{P}}^P_t, \hat{\boldsymbol{P}}^C_t, \hat{\boldsymbol{\mathcal{E}}}_t, \hat{\boldsymbol{V}}_t\}^\infty_{t_0}$ for a given sequence of $\{\hat{\boldsymbol{\tau}}_t\}^\infty_{t_0}$ and an initial condition for $\hat{\boldsymbol{V}}_0$ such that equations (4)-(8) hold:

$$\textbf{NKIS:} \quad \sigma(\mathbb{E}_t\hat{\boldsymbol{C}}_{t+1} - \hat{\boldsymbol{C}}_t) = \boldsymbol{\Phi}\boldsymbol{\Gamma}(\hat{\boldsymbol{P}}^P_t - \hat{\boldsymbol{P}}^P_{t-1}) - \mathbb{E}_t(\hat{\boldsymbol{P}}^C_{t+1} - \hat{\boldsymbol{P}}^C_t) \tag{4}$$

$$\textbf{CPI:} \quad \hat{\boldsymbol{P}}^C_t = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^C_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}^C_{\tau}\hat{\boldsymbol{\tau}}_t \tag{5}$$

$$\textbf{NKPC:} \quad \hat{\boldsymbol{P}}^P_t = \boldsymbol{\Psi}_\Lambda\left[\hat{\boldsymbol{P}}^P_{t-1} + \boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\left(\hat{\boldsymbol{P}}^C_t + \sigma\hat{\boldsymbol{C}}_t\right) + \boldsymbol{L}^P_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}^P_{\tau}\hat{\boldsymbol{\tau}}_t\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}^P_{t+1}\right] \tag{6}$$

$$\textbf{UIP:} \quad \tilde{\boldsymbol{\Phi}}_1\mathbb{E}_t\hat{\boldsymbol{\mathcal{E}}}_{t+1} - \tilde{\boldsymbol{\Phi}}_2\hat{\boldsymbol{\mathcal{E}}}_t = \tilde{\boldsymbol{\Phi}}_3\boldsymbol{\Gamma}(\hat{\boldsymbol{P}}^P_t - \hat{\boldsymbol{P}}^P_{t-1}) \tag{7}$$

$$\textbf{BoP:} \quad \beta\hat{\boldsymbol{V}}_t = \boldsymbol{\Xi}_1\hat{\boldsymbol{V}}_{t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}^P_t + \boldsymbol{\Xi}_4\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{\Xi}_5\hat{\boldsymbol{\tau}}_t + \boldsymbol{\Xi}_6\hat{\boldsymbol{P}}^P_{t-1} \tag{8}$$

where $\boldsymbol{\Psi}_\Lambda = [(1+\beta)\boldsymbol{I} + \boldsymbol{\Lambda}(\boldsymbol{I} - \boldsymbol{\Omega})]^{-1}$ is a stickiness-adjusted Leontief Inverse. We define $V_{n,t} = (1 + i^{US}_t)B^{US}_{n,t}$ and vectorize this debt variable as $\hat{\boldsymbol{V}}_t$. In the first, fourth and fifth of these equilibrium conditions, we use the Taylor rule to substitute out the nominal interest

[11] In particular, $(\boldsymbol{L}^P_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t)_{ni} = \sum_{m\in N}\sum_{j\in J}\Omega_{ni,mj}\hat{\mathcal{E}}_{n,m,t}$ and $(\boldsymbol{L}^P_{\tau}\hat{\boldsymbol{\tau}}_t)_{ni} = \sum_{m\in N}\sum_{j\in J}\Omega_{ni,mj}\hat{\tau}_{n,mj,t}$.

[12] Expressing prices in levels rather than first differences yields a compact five-variable, five-equation system and is convenient for MUC algebra.

rate, where the diagonal matrix $\mathbf{\Phi}$ contains the Taylor rule's sensitivity to the basket of producer price inflation in the respective countries. That is, we have $\hat{\boldsymbol{i}}_t = \mathbf{\Phi\Gamma}(\hat{\boldsymbol{P}}_t^P - \hat{\boldsymbol{P}}_{t-1}^P)$ and the first $N-1$ rows of $\tilde{\mathbf{\Phi}}_3\mathbf{\Gamma}(\hat{\boldsymbol{P}}_t^P - \hat{\boldsymbol{P}}_{t-1}^P)$ load the vector form of interest rate differentials $\hat{\boldsymbol{i}}_t - \hat{i}_t^{US}$ for countries other than the first country in our system, the U.S. Relatedly, the last equation features $\hat{\boldsymbol{P}}_{t-1}^P$, because the interest rate is substituted out.

The first of these equilibrium conditions is the Euler (New Keynesian IS; NKIS) equation, which is defined in terms of aggregate consumer prices. Intuitively, the impact of tariffs enters the demand side through how tariffs load onto consumer prices.

The second equation defines the consumer price index (CPI). Here $\mathbf{\Gamma}$ is an $N \times NJ$ matrix, containing consumption shares[13] and $\boldsymbol{L}_{\mathcal{E}}^C$ captures, in matrix form, how consumer prices of various goods are exposed to the exchange rate. The scalar analogy would be $(1-\gamma_H)$, where $\gamma_H \in [0,1]$ represents the home bias parameter for consumption. Similarly, $\boldsymbol{L}_{\tau}^C$ captures the share of goods exposed to tariffs.

The third equation is the New Keynesian Phillips Curve for producer price inflation, defined in levels for analytical convenience. The stickiness-adjusted Leontief inverse $\mathbf{\Psi}_\Lambda$ captures the I-O network and multiplies $\mathbf{\Lambda}$, nominal marginal costs, lagged prices $\hat{\boldsymbol{P}}_{t-1}^P$, and discounted expected prices $\beta\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P$. Exchange rates load onto marginal costs through imported-input dependence ($\boldsymbol{L}_{\mathcal{E}}^P$), and tariffs through the share of tariff-exposed goods ($\mathbf{L}_{\tau}^P$).

The fourth equation combines the UIP condition, exchange rate arbitrage conditions, and the definition of a country's exchange rate with itself. Here, the $\tilde{\mathbf{\Phi}}$ terms ensure that the ϕ_π terms for each country, along with the arbitrage conditions, are loaded in each row.

The fifth equation combines debt-market clearing with the $N-1$ laws of motion for net debt, expressing the balance of payments as a function of prices—which reflect good-specific terms of trade—and the aggregate consumption vector.[14] This final equation describes how a country's net external position evolves in response to changes in good-specific terms of trade, as well as fluctuations in the interest rate and the balance sheet effect of debt via exchange rates. As such, it nests all the intratemporal relative demand conditions and pricing equations summarized by the $\mathbf{\Xi}$ coefficients, which we derive in detail in Appendix B. Through this equation, debt responds to adjustments in net exports following changes in the terms of trade, the balance sheet effect and interest payments.

This five-equation representation nests a broad class of open-economy New Keynesian models. Models with intermediate inputs and a final good correspond to $J = 2$ with one column of $\mathbf{\Omega}$ set to zero. Collapsing to a single country with an exogenous real rate recovers

[13] Similar to the production case, $(\boldsymbol{L}_{\mathcal{E}}^C\hat{\boldsymbol{\mathcal{E}}}_t)_n = \sum_{m\in N}\sum_{j\in J}\Gamma_{n,mj}\hat{\mathcal{E}}_{n,m,t}$ and $(\boldsymbol{L}_{\tau}^C\hat{\boldsymbol{\tau}}_t)_n = \sum_{m\in N}\sum_{j\in J}\Gamma_{n,mj}\hat{\tau}_{n,mj,t}$.

[14] The first $N-1$ rows contain linearized versions of equation (2), while the last row captures the bond market clearing condition In Appendix B, we derive this equation of motion.

the canonical small open economy model reminiscent of Galí and Monacelli (2005).

2.5 Wealth Transfers Summarized By the Risk-Sharing Wedge

From here through Section 4, we focus on the two-country case, $N = 2$, to build intuition, with H denoting the tariff-imposing home country and F the foreign country. Hence, bilateral variables, including $\hat{\mathcal{E}}_t$, are scalars; without loss of generality, so is the tariff, $\hat{\tau}_t$. Section 5 returns to the general case.

Under incomplete markets, tariffs facilitate a wealth transfer between countries that can be summarized by a risk-sharing wedge, $\hat{w}_t$ that captures deviations from the perfect risk sharing benchmark. We find this object useful for two reasons. First, substantively, it summarizes the wealth transfer to home (foreign) that results from terms of trade gains (losses) and the balance sheet effects of improved (deteriorated) net debt position of countries (can be both nominal and real via home currency appreciation (depreciation) and/or smaller (larger) trade deficit). Secondly, in our method of undetermined coefficients, as we detail below, it will be more convenient to track this object as a state variable rather than the net debt variable, $\hat{V}_t$.

To understand the intuition behind $\hat{w}_t$ let us begin with the perfect risk-sharing benchmark. Under complete markets (e.g., in the presence of Arrow-Debreu securities) perfect risk sharing implies the following standard, linearized Backus Smith condition: $\sigma\big(\hat{C}_{H,t} - \hat{C}_{F,t}\big) = \hat{Q}_t$, where $\hat{Q}_t \equiv \hat{P}^C_{F,t} + \hat{\mathcal{E}}_t - \hat{P}^C_{H,t}$ is the real exchange rate.[15] With ex-ante full insurance, the allocation is implemented through state-contingent transfers rather than a single nominal bond, so the planner reallocates resources to where a unit of consumption yields higher marginal utility. When $\hat{Q}_t$ is high – the home consumption basket is relatively cheap in common currency – efficiency requires higher home consumption, and capital flows toward the home country to ensure relative consumption exceeds its steady-state level. Under incomplete markets, households cannot trade state-contingent claims and instead borrow and lend internationally through a single non-state-contingent bond (the U.S. bond). Combining the Euler equation with UIP shows that under incomplete markets, the Backus–Smith condition holds only *in expectation*: $\sigma(\mathbb{E}_t\Delta\hat{C}_{H,t+1} - \mathbb{E}_t\Delta\hat{C}_{F,t+1}) = \mathbb{E}_t\Delta\hat{Q}_{t+1}$. Defining $\hat{w}_t \equiv \hat{Q}_t - \sigma(\hat{C}_{H,t} - \hat{C}_{F,t})$, this can be written as $\hat{w}_t = \mathbb{E}_t\hat{w}_{t+1}$, so the risk-sharing wedge is a martingale.[16]

[15] This condition follows from the complete-markets optimality condition that equates state-contingent marginal utilities, when valued in the same currency: $\frac{U_C(C_{H,t})}{P^C_{H,t}} = \lambda \frac{U_C(C_{F,t})}{\mathcal{E}_t P^C_{F,t}}$ for a constant λ pinned down by initial wealth. Linearizing this condition around a steady-state under CRRA utility yields $\sigma(\hat{C}_{H,t} - \hat{C}_{F,t}) = \hat{Q}_t$.

[16] The martingale property relies on zero portfolio adjustment costs in our analytical work. Because these costs are small, including them produces near-martingale behavior without materially altering our conclusions.

For a tariff shock $\hat{\tau}_t$ that reveals itself at $t = 0$ and decays with persistence ρ_τ, $\hat{w}_t$ is a linear function of the initial shock: $\hat{w}_t = f(\hat{\tau}_0)\ \forall t$. Because the shock is unexpected and cannot be insured ex ante, it acts as a wealth transfer when revealed. If the terms-of-trade gains and balance sheet effects favor the home country, $\hat{w}_t$ will be negative for standard parameterizations of θ, so wealth transfers to home country, generating appreciationary pressure and pushing home consumption above the perfect risk-sharing benchmark.[17]

2.6 Tariffs Transmission and Propagation Channels

Since $\hat{w}_t = \mathbb{E}_t \hat{w}_{t+1}$ is a martingale, we can divide the five-equation representation into two blocks. In the New Keynesian block, we treat the risk-sharing wedge as a state variable along with tariffs and substitute out the exchange rate using the definition of the risk-sharing wedge: $\hat{\mathcal{E}}_t = \hat{w}_t - \hat{P}^C_{F,t} + \hat{P}^C_{H,t} + \sigma\big(\hat{C}_{H,t} - \hat{C}_{F,t}\big)$. We can then write the New Keynesian block as follows:[18]

$$\textbf{NKIS:} \qquad \sigma \mathbb{E}_t \Delta \hat{\boldsymbol{C}}_{t+1} = \underbrace{(\boldsymbol{I} - \boldsymbol{L}^C_{\mathcal{E}} \boldsymbol{Z}) \boldsymbol{\Phi} \boldsymbol{\Gamma} \boldsymbol{\pi}^P_t - \boldsymbol{\Gamma}\, \mathbb{E}_t \boldsymbol{\pi}^P_{t+1} - \boldsymbol{L}^C_\tau\, \mathbb{E}_t \Delta \hat{\tau}_{t+1}}_{\hat{\boldsymbol{i}}_t - \mathbb{E}_t \boldsymbol{\pi}^C_{t+1}} \tag{9}$$

$$\textbf{NKPC:} \qquad \boldsymbol{\pi}^P_t = \boldsymbol{\Lambda} \underbrace{\boldsymbol{\mu}_t}_{\text{Real Marginal Cost (RMC)}} + \beta\, \mathbb{E}_t \boldsymbol{\pi}^P_{t+1} \tag{10}$$

$$\textbf{RMC} \qquad \boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} + \boldsymbol{\mu}_1\, \boldsymbol{\pi}^P_t + \boldsymbol{\mu}_2 \sigma \Delta \hat{\boldsymbol{C}}_t + \left(\boldsymbol{\mu}_2 \boldsymbol{L}^C_\tau + \boldsymbol{L}^P_\tau\right) \Delta \hat{\tau}_t + \boldsymbol{\mu}_4 \Delta \hat{w}_t \tag{11}$$

where Δ notation indicates first difference, $\boldsymbol{Z} = \begin{bmatrix} 1 - 1 \end{bmatrix}$ differences home and foreign entries, $\boldsymbol{\mu}_1 \equiv \boldsymbol{\Omega} - \boldsymbol{I} + \boldsymbol{\alpha} \boldsymbol{\Gamma} + (\boldsymbol{\alpha} \boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{L}^P_{\mathcal{E}}) m \boldsymbol{Z} \boldsymbol{\Gamma}$, capturing the impact of producer prices via input-output linkages, wages and the exchange rate, $\boldsymbol{\mu}_2 \equiv \Big(\boldsymbol{\alpha} + (\boldsymbol{\alpha} \boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{L}^P_{\mathcal{E}}) m \boldsymbol{Z}\Big)$, capturing the impact of consumption and tariffs on real wages and the exchange rate, and $\boldsymbol{\mu}_4 \equiv (\boldsymbol{\alpha} \boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{L}^P_{\mathcal{E}}) m$ where $m \equiv \left(1 - \boldsymbol{Z} \boldsymbol{L}^C_{\mathcal{E}}\right)^{-1}$ capturing the impact of the risk-sharing wedge via the exchange rate's impact on real wages and producer prices.

As the New-Keynesian block above demonstrates: (i) Non-transitory tariffs induce a demand shock (e.g., a patience shock). Tariffs are a tax on the consumption; as such they are a *relative* demand shock. In the absence of perfect substitutability, however, this consump-

[17] Other shocks can also move $\hat{w}_t$: in Section 5, we allow an additional shock on top of tariffs, that is tariff uncertainty. This second shock raises the UIP premium, as in Kalemli-Özcan et al. (2026), leading to U.S. dollar depreciation and turning $\hat{w}_t > 0$, depressing home consumption with wealth transfers to foreign.

[18] In the full model, the lagged price vector is a state variable because the input-output structure makes *the rate of change of prices*, depend on the full vector of the *level of producer prices*. For ease of interpretation, instead of the vector of lagged producer prices, $\hat{\boldsymbol{P}}^P_{t-1}$ we track the vector of lagged real marginal costs in each sector, $\boldsymbol{\mu}_{t-1}$, as a state variable in the New Keynesian block, as the latter allows us to track deviations from the flexible-price benchmark, which constitute a distortion across time.

tion tax has a direct effect on the aggregate consumption price index and acts similarly to an aggregate consumption tax. We denote this direct effect with $\boldsymbol{L}^C_\tau$. This shock makes consumption more expensive in some periods relative to others, thereby impacting both intertemporal substitution and the household's labor supply as other demand shocks do.[19] Because it distorts the household's labor supply, $\boldsymbol{L}^C_\tau$ also impacts real marginal cost, $\boldsymbol{\mu}_t$. (ii) Tariffs are a tax placed on intermediate inputs and hence they are a supply shock that makes it more expensive for the industries of the tariff-imposing country to produce. We denote this direct effect on the real marginal cost basket with $\boldsymbol{L}^P_\tau$. (iii) As noted in Section 2.5 tariffs constitute a wealth transfer between countries, summarized by the risk-sharing wedge. If the wealth transfer via the risk-sharing wedge favors the home country, then it can partially or more than offset the increase in $\boldsymbol{\mu}_t$ from tariffs via exchange rate appreciation.

Remark 1. The wealth transfer captured by the risk-sharing wedge impacts the inflation-output tradeoff.[20] When the wedge is negative (positive), and thus favors the tariff-imposing country (tariffed country), it can increase (decrease) output and decrease (increase) inflation, offsetting (amplifying) the negative supply shock impact of tariffs that is stagflationary.

After solving the New Keynesian block, treating $\hat{w}_t$ as a state variable, we solve the open-economy block, comprising the martingale equation $\hat{w}_t = \mathbb{E}_t \hat{w}_{t+1}$ (which replaces the UIP), the balance of payments equation, and the necessary variable definitions, using the coefficients from the first block to pin down $\hat{w}_t$ and $\hat{V}_t$. There is a unique level of $\hat{w}_t$ that satisfies the martingale condition and ensures that $\hat{V}_t$ is not on an explosive path.[21] This two-block approach – solving in terms of the risk-sharing wedge and then determining the wedge from the remaining equilibrium conditions – is a methodological contribution to production network models with incomplete markets and to general open-economy models.

3 Tariffs Under Flexible Prices

We first study the flexible-price two-country, one-good economy ($N = 2$, $J = 1$), and then extend the analysis to multiple sectors to isolate the network channel. Setting $J = 1$, we consider unilateral tariffs so the foreign entries of $\boldsymbol{L}^C_\tau$ and $\boldsymbol{L}^P_\tau$ equal zero, and rule out self-use.

[19] When tariffs are permanent, instead of intertemporal substitution, there is a one-time and permanent change in all variables. Here, we focus on the case when $0 \leq \rho_\tau < 1$.

[20] Even though our model is written to track consumption as the aggregate quantity, our argument here extends to the inflation output tradeoff.

[21] We confirm numerically that solving in two blocks yields the same result as solving the full model.

The relevant matrices are:[22]

$$\mathbf{\Omega} = \begin{bmatrix} 0 & \Omega_H \\ \Omega_F & 0 \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} 1-\Omega_H & 0 \\ 0 & 1-\Omega_F \end{bmatrix}, \quad \mathbf{\Gamma} = \begin{bmatrix} 1-\gamma_H & \gamma_H \\ \gamma_F & 1-\gamma_F \end{bmatrix}, \quad \mathbf{\Psi} = (\mathbf{I}-\mathbf{\Omega})^{-1},$$
$$\boldsymbol{L}^C_{\mathcal{E}} = \begin{bmatrix} \gamma_H \\ -\gamma_F \end{bmatrix}, \quad \boldsymbol{L}^C_{\tau} = \begin{bmatrix} \gamma_H L^C_\tau \\ 0 \end{bmatrix}, \quad \boldsymbol{L}^P_{\mathcal{E}} = \begin{bmatrix} \Omega_H \\ -\Omega_F \end{bmatrix}, \quad \boldsymbol{L}^P_{\tau} = \begin{bmatrix} \Omega_H L^P_\tau \\ 0 \end{bmatrix}$$

Here, L^C_τ and L^P_τ are indicators for whether one country imposes tariffs on the other. Subscripts H and F denote home and foreign. Under symmetry, we set $\Omega_H = \Omega_F = \Omega$ and $\gamma_H = \gamma_F = \gamma$, with $0 \leq \gamma < \frac{1}{2}$ and $0 \leq \Omega < 1$. In this notation, consider the symmetric one-good economy under flexible prices and assume monetary policy stabilizes aggregate prices rather than following a Taylor rule.[23] Setting $\sigma = 1$, the equilibrium implied by the New Keynesian block (i.e., $\Lambda_{ni} \to \infty$ for all $n \in N, i \in J$), taking $\hat{w}_t$ as a state variable, is defined by:

Definition 2. An approximate flexible-price equilibrium is a sequence $\{\hat{P}^P_{H,t}, \hat{P}^P_{F,t}, \hat{\mathcal{E}}_t, \hat{i}_{H,t}, \hat{i}_{F,t}, \hat{C}_{H,t}, \hat{C}_{F,t}\}_{t=0}^{\infty}$ such that, for exogenous $\{\hat{\tau}_t, \hat{w}_t\}_{t=0}^{\infty}$:

1. Euler equations hold: $(\mathbb{E}_t\hat{C}_{H,t+1} - \hat{C}_{H,t}) = \hat{i}_{H,t}$ and $(\mathbb{E}_t\hat{C}_{F,t+1} - \hat{C}_{F,t}) = \hat{i}_{F,t}$.
2. Basket prices satisfy $0 = (1-\gamma)\hat{P}^P_{H,t} + \gamma(\hat{\mathcal{E}}_t + \hat{P}^P_{F,t} + \hat{\tau}_t)$ and $0 = (1-\gamma)\hat{P}^P_{F,t} + \gamma(\hat{P}^P_{H,t} - \hat{\mathcal{E}}_t)$.
3. Prices equal marginal cost: $\hat{P}^P_{H,t} = (1-\Omega)\hat{C}_{H,t} + \Omega(\hat{P}^P_{F,t} + \hat{\tau}_t + \hat{\mathcal{E}}_t)$ and $\hat{P}^P_{F,t} = (1-\Omega)\hat{C}_{F,t} + \Omega(\hat{P}^P_{H,t} - \hat{\mathcal{E}}_t)$.
4. Risk sharing is imperfect: $\hat{\mathcal{E}}_t - (\hat{C}_{H,t} - \hat{C}_{F,t}) = \hat{w}_t$.

It helps to present an equilibrium relationship linking the risk-sharing wedge and the terms of trade. Defining the terms of trade $\hat{s}_t \equiv \hat{\mathcal{E}}_t + \hat{p}_{F,t} - \hat{p}_{H,t}$, the intratemporal conditions give:

$$\hat{w}_t = \frac{1+\Omega}{1-\Omega}\hat{s}_t + \frac{\Omega}{1-\Omega}L^P_\tau \hat{\tau}_t.$$

The wedge thus embeds the terms-of-trade gains from tariff imposition (by a large country) that the trade literature emphasizes in optimal-tariff analysis. Without intermediate inputs, elastic labor implies $\hat{w}_t = \hat{s}_t$. With intermediate inputs, tariffs on production inputs can raise the wedge and depreciate the home currency. Although the wedge grows more complex once these assumptions are relaxed, the central mechanism is clear: terms-of-trade movements favoring the tariff-imposing country transfer wealth to it, allowing consumption

[22] In this scalar case, we simplify subscripts: $\gamma_{H,F}$ becomes γ_H and $\Omega_{H,F}$ becomes Ω_H.
[23] Under flexible prices this leaves real allocations unchanged and simplifies notation.

above the perfect-risk-sharing benchmark. If the home country is highly dependent on foreign intermediates, this can reverse: terms of trade can move against it, transferring wealth abroad.

Proposition 1. *Under symmetry and flexible prices, solving the model yields*

$$\hat{C}_{H,t} = -\underbrace{\frac{\Omega(1-\gamma)+\gamma}{1+\Omega}}_{>0}\left(\frac{1}{1-\Omega}\hat{\tau}_t + \hat{w}_t\right) \tag{12}$$

$$\hat{\mathcal{E}}_t = -\underbrace{\frac{(\Omega(1-\gamma)+\gamma)}{1+\Omega}}_{>0}\hat{\tau}_t + \underbrace{\frac{(1-\Omega)(1-2\gamma)}{1+\Omega}}_{>0}\hat{w}_t \tag{13}$$

Proposition 1 follows from MUC algebra; Appendix C contains the derivation. Two immediate corollaries follow.

Corollary 1. *Under perfect risk sharing ($\hat{w}_t = 0$), a unilateral home import tariff lowers home consumption and appreciates the home currency ($\frac{\partial \hat{C}_t}{\partial \hat{\tau}_t} < 0, \frac{\partial \hat{\mathcal{E}}_t}{\partial \hat{\tau}_t} < 0$).*

Tariffs shift demand toward domestic goods. With home bias, the home consumption basket becomes more expensive and the home currency appreciates in real terms. Under perfect risk sharing, Arrow-Debreu transfers reallocate resources toward states where home consumption is relatively cheap, lowering contemporaneous home consumption. The decline in the tariff-imposing country's consumption in frameworks such as Caliendo et al. (2025) reflects this mechanism.

Corollary 2. *For tariffs to raise home consumption, tariffs must induce a deviation from perfect risk sharing (i.e., $\hat{w}_t \neq 0$), thereby producing a wealth transfer towards home in states when consumption is expensive.*

Under imperfect risk sharing, the fall in consumption generated by intertemporal substitution (consumption is expensive today with tariffs) can be offset by the permanent wealth effect embodied in the martingale wedge. In equilibrium, consumption rises if the wedge is sufficiently negative. Consumption falls with a sufficiently positive wedge that also implies depreciation and a wealth transfer to the rest of the world, strengthening foreign consumption and appreciating the foreign currency. Under incomplete markets, the sign and magnitude of $\hat{w}_t$ can therefore change the sign and magnitude of consumption and exchange rate. The tariff-induced wedge that opens on impact and remains constant thereafter is ($\forall\ k \geq 0$):

$$\hat{w}_{t+k} = \underbrace{\left[\frac{1-\beta}{1-\beta\rho_\tau}\right]}_{>0} \frac{\overbrace{\mathcal{A}\left(\gamma(1-\Omega)+\frac{2\Omega}{1-\Omega}\right)+\theta\left[\Omega+\gamma(1-\gamma)(1-\Omega)^2\right]}^{>0}}{\underbrace{\mathcal{A}(1-2\gamma)(1-\Omega)}_{>0}-\underbrace{2\theta\left[\Omega+\gamma(1-\gamma)(1-\Omega)^2\right]}_{>0}} \tag{14}$$

where $\mathcal{A} \equiv \gamma + (1-\gamma)\Omega > 0$. Appendix C provides details for Equation (14). In this expression, the persistence-discount prefactor and numerator are strictly positive, so the denominator determines the sign. The threshold is:

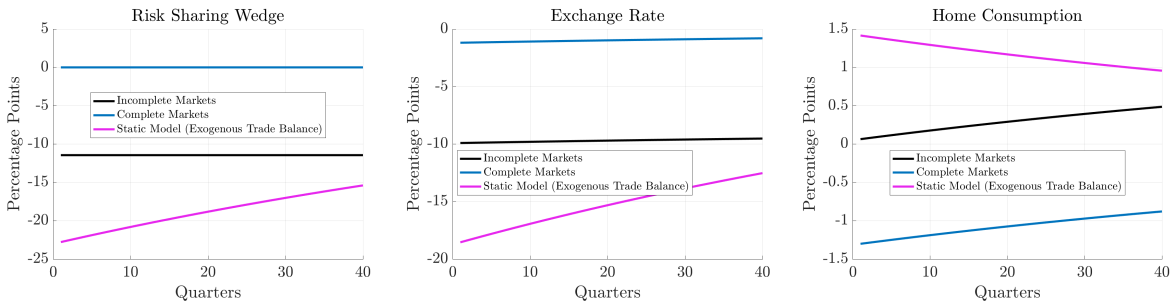
$$\theta^{\text{crit}} \equiv \tfrac{\mathcal{A}(1-2\gamma)(1-\Omega)}{2[\Omega+\gamma(1-\gamma)(1-\Omega)^2]}, \text{ where } \hat{w}_t > 0 \text{ for } \theta < \theta^{\text{crit}} \text{ and } \hat{w}_t < 0 \text{ for } \theta > \theta^{\text{crit}}.$$

Greater substitutability makes it more likely that terms-of-trade gains favor the home country, transferring wealth to home, increasing consumption at home and appreciating the home currency (a sufficiently negative wedge). The magnitude of the wedge is also sensitive to tariff persistence (ρ_τ) as persistent tariffs lead to larger wealth effects.

Remark 2. Greater tariff persistence enlarges the risk-sharing wedge in absolute value: terms-of-trade gains then last longer and imply a larger wealth transfer. While primitives such as $\mathbf{\Gamma}$, $\mathbf{\Omega}$, and θ can reverse the sign of $\hat{w}_t$, persistence can also make the wedge large enough to flip the signs of the expressions in (12) and (13).

To highlight the importance of imperfect risk sharing, we compare incomplete markets with two standard alternatives: complete markets and exogenous transfers, as in static trade models that pin down the trade balance. Figure 1 reports the wedge in panel (a), the exchange rate in panel (b), and home consumption in panel (c) after a 10% home tariff with $\rho_\tau = 0.99$, $\gamma = 0.05$, and $\Omega = 0.1$. Under complete markets (blue), the wedge is zero. Under incomplete markets (black), it is permanently negative. Under exogenous transfers (pink), it is also negative but larger in absolute value, as eliminating financial assets creates a greater departure from perfect risk sharing. In the static model, the wedge declines in absolute value over time, reflecting the fact that it is not a martingale: there, $\hat{w}_t$ depends only on contemporaneous tariffs and therefore reverts to zero as tariffs decline. The exchange rate appreciates in all cases, with negligible appreciation under complete markets and a large one in the static transfer case. The home-consumption response is positive in both the static transfer and incomplete-markets cases, but negative under complete markets. With financial assets, consumption depends on the wedge (as in (12)): when $\hat{w}_t = 0$, perfect risk sharing implies a negative consumption response, whereas a sufficiently negative wedge lets

Figure 1. Comparing Complete Markets, Incomplete Markets, and the Static Model



NOTE: Panel (a) plots the risk-sharing wedge, panel (b) the exchange rate, and panel (c) home consumption after a 10% home tariff on the rest of the world with persistence $\rho_\tau = 0.99$. We set $\gamma = 0.05$ and $\Omega = 0.1$. Black denotes incomplete markets, blue complete markets, and pink the static exogenous-transfer specification that fixes the trade balance at its steady-state level. Under complete markets the wedge is zero; under incomplete markets and the static specification it is negative, so consumption and exchange-rate responses differ across cases.

the intertemporal wealth effect dominate substitution and raise consumption, as in panel (c). Consistent with our theory, the large absolute value of $\hat{w}_t$ is partly driven by the high tariff persistence ($\rho_\tau = 0.99$).

Figure 1 therefore shows that neither complete markets nor static exogenous transfers constitute innocuous benchmarks. Complete markets impose $\hat{w}_t = 0$, shutting down tariff-induced wealth transfers and thereby making the home-consumption response negative by construction. Static models move in the opposite direction and can overstate the wedge since the trade balance is unable to adjust endogenously, which can generate an unrealistically large appreciation; in this calibration, the appreciation response is nearly twice the size of the tariff. Because $\hat{w}_t$ enters both the consumption response and the inflation–output tradeoff, some form of incomplete markets—at minimum, a single non-state-contingent nominal bond—is essential to understanding the macroeconomic impact of tariffs.

3.1 N Countries and J Sectors: Why the Network Matters?

We next examine how production networks affect tariff transmission under flexible prices. When intermediate inputs are absent (i.e., $\Omega = 0$), (14) reduces to ($\forall\ k \geq 0$):

$$\hat{w}_{t+k} = \left[\frac{1-\beta}{1-\beta\rho_\tau}\right] \frac{\gamma + \theta(1-\gamma)}{(1-2\gamma) - 2\theta(1-\gamma)}.$$

Thus, without intermediate inputs ($\Omega = 0$), $\theta > \frac{(1-2\gamma)}{2(1-\gamma)}$ is sufficient for the risk-sharing wedge to be negative, favoring the tariff-imposing home country. This threshold is satisfied under most calibrations in the literature, as θ often exceeds 0.5. Once $\Omega \neq 0$, however, the sign of the wedge can vary over a wider range.

To illustrate, let us add a second foreign good, denoted T. It is upstream, does not enter final consumption, is produced solely with labor, and is used by the home country as an intermediate input (its share denoted by Ω_{HT}). Then the risk-sharing wedge is ($\forall\ k \geq 0$):

$$\hat{w}_{t+k} = -\frac{1-\beta}{1-\beta\rho_\tau}\,\frac{(\Omega+\Omega_{H,T})\Big[\Omega\Big(\Omega_{H,T}y(2-\theta^X)-\beta\Big)+\alpha_F(A_2-\beta)\Big]-A_1\,A_3}{A_1\Big(1-y\mathcal{D}\,\alpha_H\Big)-\alpha_H\Big[\Omega\,\Omega_{H,T}y(2-\theta^X)+\alpha_F A_2\Big]},$$

where $\mathcal{D} \equiv \frac{1}{1+\Omega}$, $A_1 \equiv \Omega\Omega_{H,T}+\Omega\alpha_F+\Omega\alpha_H+\Omega_{H,T}\alpha_F+\alpha_F\alpha_H > 0$, $A_2 \equiv y\Big(1+\Omega_{H,T}+\mathcal{D}\big[-\alpha_H-\theta^X(2\Omega+\Omega_{H,T})\big]\Big)$, and $A_3 \equiv -y^2\mathcal{D}\,\theta^X L^P_\tau\big(\Omega\alpha_F+\Omega_{H,T}\big) < 0$.

When θ is low, that is inputs are highly complementary, the sign of $\hat{w}_{t+k}$ can reverse with Ω_{HT}. Tariffs on the upstream good make the home good a scarce input to foreign production as foreign production also uses it as an input, the resulting relative-supply and -demand shifts can make foreign goods more valuable, turning the wedge positive and favoring the foreign country.

4 Tariffs Under Sticky Prices

We now reintroduce nominal rigidities and monetary policy and thereby turn to the analytical solution for our model under sticky prices. In the $N = 2$ & $J = 1$, case the primitives we are adding correspond to the following matrices and scalar objects: $\mathbf{\Lambda} = \begin{bmatrix}\Lambda_H & 0\\ 0 & \Lambda_F\end{bmatrix}$, $\mathbf{\Phi} = \begin{bmatrix}\phi^H_\pi & 0\\ 0 & \phi^F_\pi\end{bmatrix}$. With that, we return to the system of equations in (9)-(11) which can be solved with the MUC as follows:

$$\begin{aligned}\Delta\hat{\boldsymbol{C}}_t &= \boldsymbol{c}_\mu\boldsymbol{\mu}_{t-1}+\boldsymbol{c}_w\Delta\hat{w}_t+\boldsymbol{c}_\tau\hat{\tau}_t+\boldsymbol{c}_{\tau,-1}\hat{\tau}_{t-1},\\ \boldsymbol{\pi}^P_t &= \boldsymbol{p}_\mu\boldsymbol{\mu}_{t-1}+\boldsymbol{p}_w\Delta\hat{w}_t+\boldsymbol{p}_\tau\hat{\tau}_t+\boldsymbol{p}_{\tau,-1}\hat{\tau}_{t-1},\\ \boldsymbol{\mu}_t &= \mathbf{\Psi}^{\mathrm{NKOE}}\boldsymbol{\mu}_{t-1}+\boldsymbol{\mu}_w\Delta\hat{w}_t+\boldsymbol{\mu}_\tau\hat{\tau}_t+\boldsymbol{\mu}_{\tau,-1}\hat{\tau}_{t-1},\end{aligned}$$

Our first step is to introduce the NKOE propagation matrix, $\mathbf{\Psi}^{\mathrm{NKOE}}$, which is the coefficient matrix in front of the lagged real marginal cost vector in the solution for the real marginal cost vector. As such, it governs the equation of motion for real marginal costs, in deviation from the pre-tariff steady state, across time. For a candidate propagation matrix

$\boldsymbol{X}$ and tariff persistence ρ, define

$$\mathcal{K}(\rho, \boldsymbol{X}) \equiv (1-\beta\rho)\Big[\left((1-\rho)\boldsymbol{I}-\boldsymbol{X}\right)\left(\boldsymbol{I}-\beta\boldsymbol{X}\right)-(\boldsymbol{I}-\boldsymbol{\Omega})\boldsymbol{\Lambda}\boldsymbol{X}\Big]+\Big[(\boldsymbol{\alpha}+\boldsymbol{L}_{\mathcal{E}}^{P}\boldsymbol{Z})\boldsymbol{\Phi}\boldsymbol{\Gamma}-\rho(\boldsymbol{I}-\boldsymbol{\Omega})\Big]\boldsymbol{\Lambda}$$

Proposition 2. *The NKOE propagation matrix, $\boldsymbol{\Psi}^{\text{NKOE}}$, is the stable matrix that solves $\mathcal{K}(0, \boldsymbol{\Psi}^{\text{NKOE}})\, \boldsymbol{\Psi}^{\text{NKOE}} = \boldsymbol{0}$.*

Proposition 2 follows from MUC algebra (see Appendix D). Intuitively, for a candidate propagation matrix $\boldsymbol{X}$, three forces act on a cost distortion. First, $((1-\rho)\boldsymbol{I}-\boldsymbol{X})(\boldsymbol{I}-\beta\boldsymbol{X})$ captures forward-looking price adjustment and discounting. Second, $-(\boldsymbol{I}-\boldsymbol{\Omega})\boldsymbol{\Lambda}\boldsymbol{X}$ captures cost inheritance through sticky intermediate inputs: today's distortion reappears tomorrow as an input cost. Third, $\big[(\boldsymbol{\alpha}+\boldsymbol{L}_{\mathcal{E}}^{P}\boldsymbol{Z})\boldsymbol{\Phi}\boldsymbol{\Gamma}-\rho(\boldsymbol{I}-\boldsymbol{\Omega})\big]\boldsymbol{\Lambda}$ captures the stabilizing role of monetary policy and the exchange rate, alongside the continued loading of a persistent tariff onto marginal cost under price stickiness. In equilibrium, $\boldsymbol{\Psi}^{\text{NKOE}}$ is the stable propagation matrix balancing these forces.

The NKOE propagation matrix links tariff-related distortions on consumption and production to the dynamics of the real marginal cost vector. A sector central to production, whether through broad use (e.g., steel, aluminum) or downstream importance (e.g., semiconductors), carries significant weight in the standard Leontief inverse. If it also has highly flexible (or rigid) prices, i.e., a vertical (or horizontal) supply curve, the tariff's inflationary impact is amplified (or muted) through the network. Analogously to how the stickiness-adjusted Leontief inverse reweights sectors via $\boldsymbol{\Lambda}$, the NKOE propagation matrix also reweights sectors by the monetary stance of their country (e.g., through consumption and the exchange rate).

Next, we solve for the risk-sharing wedge in Proposition 3 (see Appendix D.9). As noted in Section 2.6, the wedge is the unique solution satisfying the martingale condition ($\hat{w}_t = \mathbb{E}_t\hat{w}_{t+1}$) implied by UIP and ensuring debt $\hat{V}_t$ is stable. It is the wealth transfer, relative to perfect risk sharing, required for goods and asset markets to clear at a given price sequence, including the exchange rate. This depends on how net debt responds to the terms of trade, balance sheet effects, and interest payments, which is why the balance of payments coefficients $\boldsymbol{\Xi}$ enter the solution.

Proposition 3. *The risk-sharing wedge in response to a transitory increase in tariffs ($0 < \rho < 1$) under sticky prices is as follows ($\forall k \geq 0$):*

$$\frac{\partial \hat{w}_{t+k}}{\partial \hat{\tau}_t} = w_\tau \equiv \frac{\tilde{\boldsymbol{\Xi}}_2(\boldsymbol{c}_\tau+\boldsymbol{c}_{\tau,-1})+\tilde{\boldsymbol{\Xi}}_3\boldsymbol{p}_\tau-(\tilde{\boldsymbol{\Xi}}_2\boldsymbol{c}_\mu+\tilde{\boldsymbol{\Xi}}_3\boldsymbol{p}_\mu)(\boldsymbol{\Psi}^{\text{NKOE}}-\boldsymbol{I})^{-1}\boldsymbol{\mu}_\tau}{(1-\rho)\Big[(\tilde{\boldsymbol{\Xi}}_2\boldsymbol{c}_\mu+\tilde{\boldsymbol{\Xi}}_3\boldsymbol{p}_\mu)(\boldsymbol{\Psi}^{\text{NKOE}}-\boldsymbol{I})^{-1}\boldsymbol{\mu}_w-\tilde{\boldsymbol{\Xi}}_2\boldsymbol{c}_w-\tilde{\boldsymbol{\Xi}}_3\boldsymbol{p}_w-\tilde{\Xi}_4\Big]}.$$

Proposition 3 shows that the wealth-transfer channel depends on the same propagation matrix $\mathbf{\Psi}^{\text{NKOE}}$ through $(\mathbf{\Psi}^{\text{NKOE}} - \boldsymbol{I})^{-1}$, tying the wedge directly to the dynamics governing real marginal cost distortions. With these two objects in hand, Proposition 4 solves for consumption and consumer price inflation, capturing two intuitions. First, the effect on consumption and the exchange rate depends on the tariff's impact on the risk-sharing wedge, w_τ: as in the flexible-price case, a sufficiently large negative wedge can generate simultaneous appreciation and a consumption increase. Second, the NKOE propagation matrix governs dynamics.

Proposition 4. *The first period impact of a transitory increase in tariffs* ($0 < \rho < 1$) *under sticky prices on the endogenous variables is as follows:*

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \hat{\tau}_t} = \underbrace{\left[(\boldsymbol{I} + \boldsymbol{\mathcal{Z}})\mathbf{\Gamma}\Big((1-\rho)\mathbf{\Lambda}\,\mathcal{K}(\rho, \mathbf{\Psi}^{\text{NKOE}})^{-1}\Big) + \boldsymbol{\mathcal{Z}}\sigma\boldsymbol{R}_\tau - \mathbf{\Gamma}\right]\boldsymbol{L}_\tau^P}_{\textit{NKOE propagation}} + \underbrace{\mathbf{\Gamma}\boldsymbol{L}_\tau^P}_{\textit{Direct Effects of } \boldsymbol{L}_\tau^P}$$
$$+ \underbrace{\boldsymbol{L}_\tau^C}_{\textit{Direct Effects of } \boldsymbol{L}_\tau^C} + \underbrace{\Big[(1-\sigma)\boldsymbol{\mathcal{Z}}\Big]\boldsymbol{L}_\tau^C}_{\textit{Demand Propagation}} + \underbrace{\Big[\mathbf{\Gamma}\boldsymbol{p}_w + \boldsymbol{\mathcal{Z}}\big(\sigma\boldsymbol{c}_w + \mathbf{\Gamma}\boldsymbol{p}_w\big) + \boldsymbol{L}_{\mathcal{E}}^C\, m\Big] w_\tau}_{\textit{Contribution of Wealth Transfer}}$$

$$\frac{\partial \hat{\boldsymbol{C}}_t}{\partial \hat{\tau}_t} = \underbrace{\boldsymbol{R}_\tau \boldsymbol{L}_\tau^P}_{\textit{NKOE propagation}} - \underbrace{\boldsymbol{L}_\tau^C}_{\textit{Direct Effects of } \boldsymbol{L}_\tau^C} + \underbrace{\boldsymbol{c}_w\, w_\tau}_{\textit{Contribution of Wealth Transfer}}$$

where $\boldsymbol{R}_\tau \equiv \boldsymbol{\mu}_2^\ell \left\{(1-\rho)\Big[(1-\beta\rho)(\boldsymbol{I} - \beta\mathbf{\Psi}^{\text{NKOE}}) - \boldsymbol{\mu}_1\mathbf{\Lambda}\Big]\mathcal{K}(\rho, \mathbf{\Psi}^{\text{NKOE}})^{-1} - \boldsymbol{I}\right\}$, $m \equiv \big(1 - \boldsymbol{Z}\boldsymbol{L}_{\mathcal{E}}^C\big)^{-1}$, $\boldsymbol{\mathcal{Z}} \equiv \boldsymbol{L}_{\mathcal{E}}^C\, m\, \boldsymbol{Z}$, *and* $\boldsymbol{\mu}_2^\ell \equiv (\boldsymbol{\mu}_2^\top \boldsymbol{\mu}_2)^{-1}\boldsymbol{\mu}_2^\top$. *Moreover,* $\boldsymbol{c}_w$ *and* $\boldsymbol{p}_w$ *are the coefficient matrices on* $\Delta\hat{w}_t$ *in the sticky-price solutions for* $\Delta\hat{\boldsymbol{C}}_t$ *and* $\boldsymbol{\pi}_t^P$*, respectively.*

Proposition 4 follows from MUC algebra (see Appendix D). Written this way, $\mathbf{\Psi}^{\text{NKOE}}$ enters twice: through the equilibrium propagation condition $\mathcal{K}(0, \mathbf{\Psi}^{\text{NKOE}})\mathbf{\Psi}^{\text{NKOE}} = \mathbf{0}$, and through $\mathcal{K}(\rho, \mathbf{\Psi}^{\text{NKOE}})^{-1}$ in the impact responses. The same object that governs the persistence of tariff-induced cost distortions thus maps a persistent tariff into current inflation and consumption. Proposition 4 admits a transparent decomposition: the direct demand effect, $\boldsymbol{L}_\tau^C$, is the immediate rise in consumer prices of imported goods; the direct supply effect, $\boldsymbol{L}_\tau^P$, is the immediate rise in imported input costs; propagation collects the general-equilibrium feedback through sticky prices, production linkages, and monetary policy; and the risk-sharing wedge captures wealth-transfer effects.

To illustrate, consider a simple analytical example (the full quantitative exercise is reserved for Section 5). Divide the world into the United States and the rest of the world (RoW), and suppose the United States imposes a 10% tariff on all RoW imports with persistence $\rho_\tau = 0.99$.[24] Figure 2 reports the on-impact decomposition from Proposition 4. The rise in U.S. inflation is driven primarily by the two direct channels: $\boldsymbol{L}_\tau^C$ raises CPI because the tariff directly makes imported consumption more expensive, and $\boldsymbol{\Gamma L}_\tau^P$ raises CPI because imported intermediates become costlier and pass through to consumer prices. The demand-propagation term is zero since $\sigma = 1$. The NKOE propagation term is negative: the monetary reaction to higher producer prices is strong enough to dampen inflation on impact. Intuitively, the tariff shifts wealth toward the imposing country, inducing an appreciation that lowers the domestic-currency price of imports and offsets part of the initial impulse, hence the negative wealth-transfer term for the US. For the RoW, the channel reverses: wealth flows out and inflation rises.

On the consumption side, the direct demand effect is negative: tariffs make the impact period relatively expensive, so households substitute toward cheaper periods. Imported-input tariffs affect consumption through the NKOE propagation term, which is negative, reflecting the contractionary mix of the supply shock, network spillovers, and monetary tightening. The wealth transfer toward the United States raises U.S. consumption, but not enough to overturn these negative effects. For the RoW, both wealth transfer and NKOE propagation reduce consumption.

4.1 N Countries and J Sectors: Why the Global Network Matters under Sticky Prices?

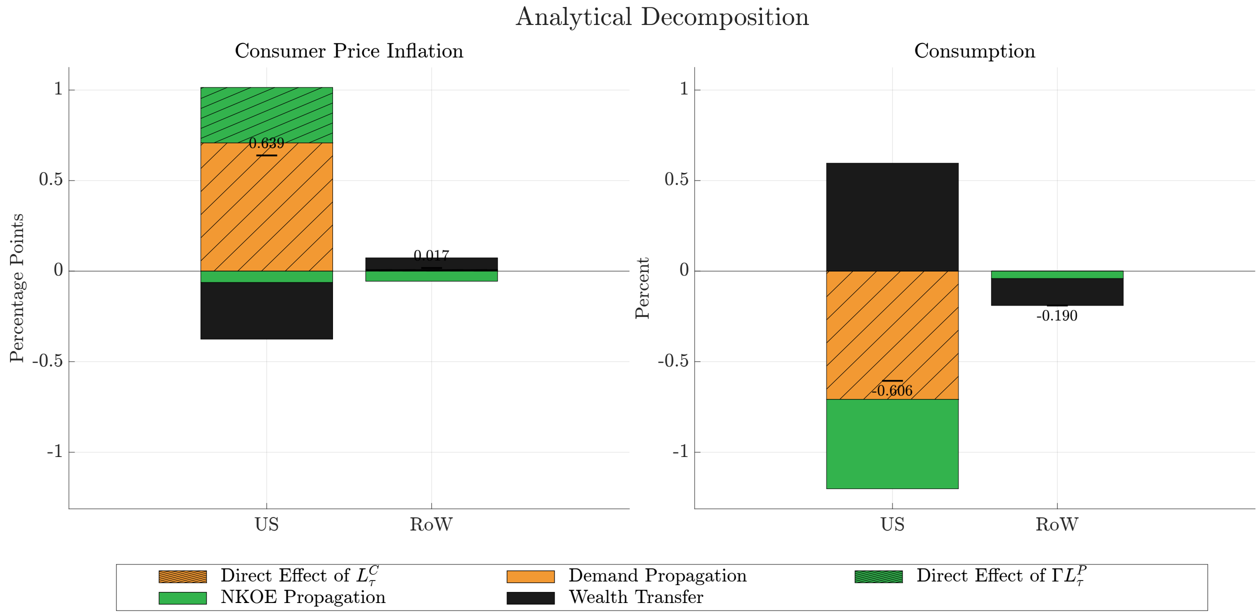
In network models, multiple sectors ($J > 1$) affect aggregate dynamics through three channels. First, under parameter heterogeneity, aggregation and multiplication do not commute: aggregating parameters before multiplying generally differs from multiplying at the sectoral level and then aggregating. Pasten et al. (2020) and Rubbo (2023) establish this result in closed economy settings; in our notation, it is reflected in Equation (3). One implication is that sectoral granularity can flatten the aggregate Phillips curve. Second, sectoral shocks interacting with sector-specific Phillips curves generate residual cost-push disturbances, so shocks to different sectors produce distinct aggregate responses. Third, and most important here, production networks make lagged deviations in real marginal costs (equivalently, lagged prices) relevant for inflation and consumption dynamics.

Proposition 5. *When $J = 1$, $\boldsymbol{\Psi}^{NKOE} = \mathbf{0}$, whereas when $J > 1$, $\boldsymbol{\Psi}^{NKOE} \neq \mathbf{0}$.*

Proof. The zero-persistence branch (i.e. $\boldsymbol{\Psi}^{\text{NKOE}} = \mathbf{0}$) requires aggregate-demand adjustments to eliminate all sectoral real marginal cost deviations on impact. In Appendix D.3,

[24] We use the parameter values in Section 5 and Table 2, except where the analytical model imposes simplifications (e.g., portfolio adjustment costs and η are set to zero, and $\sigma = 1$).

Figure 2. Theoretical Example: Tariffs Without Retaliation



NOTE: Figure 2 reports the on-impact decomposition of CPI inflation (left panel) and consumption (right panel) in a two-country analytical example with the U.S. and the rest of the world (RoW). The U.S. imposes an additional 10% tariff on the RoW with persistence $\rho_\tau = 0.99$. Using Proposition 4, the total response is decomposed into direct demand, demand propagation, direct supply, supply propagation, and wealth-transfer components. The black marker denotes the total effect.

this becomes $\boldsymbol{\mu}_2 \boldsymbol{c}_\mu = -\boldsymbol{I}_{NJ}$. When $J = 1$, $\boldsymbol{\mu}_2 \in \mathbb{R}^{N\times N}$ is square and invertible, yielding the unique solution $\boldsymbol{c}_\mu = -\boldsymbol{\mu}_2^{-1}$ and hence $\boldsymbol{\Psi}^{\text{NKOE}} = \mathbf{0}$; under Blanchard-Kahn determinacy, this is the unique equilibrium. When $J > 1$, $\boldsymbol{\mu}_2 \in \mathbb{R}^{NJ\times N}$ is tall and admits no right inverse: there are NJ sectoral distortions but only N aggregate-demand adjustments. The branch $\boldsymbol{\Psi}^{\text{NKOE}} = \mathbf{0}$ is thus impossible, so $\boldsymbol{\Psi}^{\text{NKOE}} \neq \mathbf{0}$. Appendix D.3 provides the rank argument in detail. □

Proposition 5 highlights the importance of multiple sectors in the open economy. When $J = 1$, a single sectoral price distortion per country means real marginal costs do not propagate. When $J > 1$, each country's monetary policy targets only a weighted average of prices, not the full vector of sectoral relative prices, so sectoral distortions survive aggregation and become inherited cost-push states. Hence $\boldsymbol{\mu}_t$ is a genuine state vector and the inflation–output trade-off becomes persistent. Input–output linkages then operate on this state; conditional on $J > 1$, they strengthen persistence by feeding today's sectoral cost distortions into tomorrow's marginal costs through intermediate-input use.

To understand the implications of Proposition 5, note that the solution to our model has a VAR(1) representation with the state vector defined above. Thus, the reduced-form solution can be written as $\boldsymbol{x}_t = \boldsymbol{A}\boldsymbol{x}_{t-1} + \boldsymbol{B}\,\epsilon_t$, where $\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + \epsilon_t$, $\boldsymbol{x}_t \equiv \begin{bmatrix} \boldsymbol{\mu}_t & \Delta\hat{\boldsymbol{C}}_t & \boldsymbol{\pi}_t^P & \Delta\hat{w}_t & \hat{\tau}_t \end{bmatrix}^\top$ and $\boldsymbol{B} = \begin{bmatrix} \boldsymbol{\mu}_w w_\tau + \boldsymbol{\mu}_\tau & \boldsymbol{c}_w w_\tau + \boldsymbol{c}_\tau & \boldsymbol{p}_w w_\tau + \boldsymbol{p}_\tau & w_\tau & 1 \end{bmatrix}^\top$. Here $w_\tau \equiv \frac{\partial \Delta \hat{w}_t}{\partial \epsilon_t}$, so that $\frac{\partial \Delta \hat{w}_{t+k}}{\partial \epsilon_t} = 0$ for all $j > 0$, while the level of the wedge satisfies $\frac{\partial \hat{w}_{t+k}}{\partial \epsilon_t} = w_\tau$ for all $k \geq 0$. With the VAR(1) representation, we can derive analytical impulse response functions. For example, producer price inflation, j periods after tariffs with persistence ρ_τ are imposed can be written as $\frac{\partial\, \boldsymbol{\pi}_{t+k}^P}{\partial \epsilon_t} = \boldsymbol{S}_\pi \boldsymbol{A}^k \boldsymbol{B}$ where $\boldsymbol{S}_\pi \equiv \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}$.

Remark 3. It follows from Proposition 5 that the propagation matrix $\boldsymbol{A}$ looks different when the number of sectors is $J = 1$ versus $J > 1$. When $J = 1$, Appendix D implies $\boldsymbol{\Psi}^{\text{NKOE}} = \boldsymbol{p}_\mu = \boldsymbol{p}_w = \boldsymbol{\mu}_w = \boldsymbol{0}$, while $\boldsymbol{c}_\mu = -\boldsymbol{\mu}_2^{-1}$, $\boldsymbol{c}_{\tau,-1} = \boldsymbol{\mu}_2^{-1}\boldsymbol{\mu}_3$, $\boldsymbol{p}_{\tau,-1} = \boldsymbol{0}$, and $\boldsymbol{\mu}_{\tau,-1} = \boldsymbol{0}$. Thus

$$\boldsymbol{A}_{J=1} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \rho_\tau\, \boldsymbol{\mu}_\tau \\ -\boldsymbol{\mu}_2^{-1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\mu}_2^{-1}\boldsymbol{\mu}_3 + \rho_\tau\, \boldsymbol{c}_\tau \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \rho_\tau\, \boldsymbol{p}_\tau \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & 0 & 0 \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & 0 & \rho_\tau \end{bmatrix} \text{ and}$$

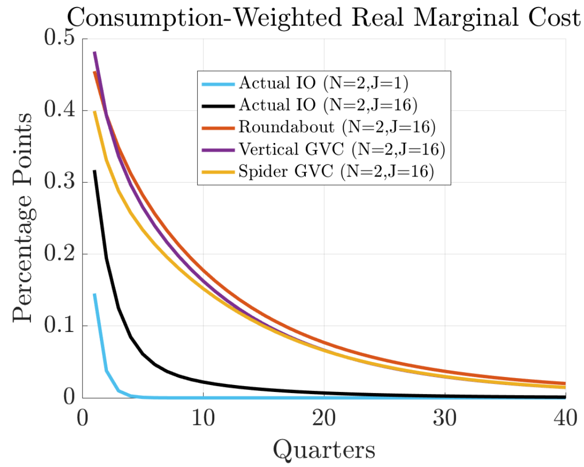
$$\boldsymbol{A}_{J>1} = \begin{bmatrix} \boldsymbol{\Psi}^{\text{NKOE}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \rho_\tau\, \boldsymbol{\mu}_\tau \\ \boldsymbol{c}_\mu & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{c}_{\tau,-1} + \rho_\tau\, \boldsymbol{c}_\tau \\ \boldsymbol{p}_\mu & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \rho_\tau\, \boldsymbol{p}_\tau \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & 0 & 0 \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & 0 & \rho_\tau \end{bmatrix}.$$

Thus, J changes how endogenous lagged variables propagate shocks. When $J = 1$, $\boldsymbol{\Psi}^{\text{NKOE}} = \boldsymbol{0}$ and lagged real marginal cost deviations have no effect on contemporaneous inflation and consumption. When $J > 1$, $\boldsymbol{\Psi}^{\text{NKOE}} \neq \boldsymbol{0}$ and the lagged real marginal cost vector generates persistence in the inflation–consumption (similarly, inflation–output) trade-off: with more granular global networks ($J > 1$), tariff-induced real marginal cost deviations take longer to clear.

Figure 3 illustrates this mechanism for a 10% tariff imposed by the US on the rest of the world under passive monetary policy, implemented as $1 + \hat{i}_{n,t} = (1 + i_{n,t-1}) + \phi_\pi^n \pi_{n,t} \quad \forall n \in N$ with $\phi_\pi^n \to 0$.[25] Each line reports CPI-weighted real marginal cost in the tariff-imposing country. Across scenarios, aggregate home bias and aggregate intermediate-input dependence are held fixed, so all networks collapse to the same 2×2 matrices $\boldsymbol{\Gamma}$ and $\boldsymbol{\Omega}$; only granular between-sector flows differ. The light blue line is the $J = 1$ case; the remaining lines use $J = 16$. The figure demonstrates that relative to the one-sector benchmark, $J = 16$ cases see

[25] Taking the limit preserves determinacy.

Figure 3. Network Granularity and Persistence of Real Marginal Cost Deviations



Note: The figure plots CPI-weighted real marginal cost in the tariff-imposing country across network structures. All granular networks aggregate to the same 2×2 matrices $\mathbf{\Gamma}$ and $\mathbf{\Omega}$. Relative to $J = 1$, economies with $J > 1$ exhibit slower decay, and persistence varies across granular networks.

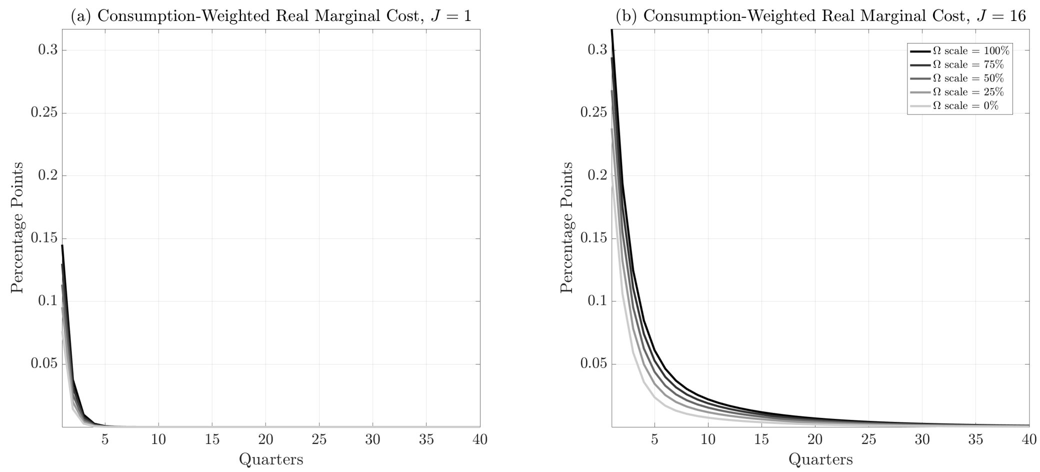
higher persistence of real marginal cost deviations across time and different network setups (e.g. roundabout and vertical and spider global value chains) produce different persistence.

The effect is not purely dimensional. Proposition 5 identifies that, once $J > 1$, inherited real marginal cost becomes a propagating state even without input–output linkages. A separate question is what $\mathbf{\Omega}$ adds once propagation is on. Since $\mathbf{\Psi}^{\text{NKOE}}$ is the coefficient on lagged real marginal cost in the solution, a larger eigenvalue governing the slowest decay implies slower unwinding of inherited marginal-cost distortions. Holding J, $\mathbf{\Gamma}$, and $\mathbf{\Lambda}$ fixed, a larger $\mathbf{\Omega}$ raises the share of costs inherited through intermediate inputs. Proposition 6 gives sufficient conditions under which this increases persistence: $J > 1$ makes real marginal cost deviations persistent, and $\mathbf{\Omega}$ governs the strength of that persistence.

Proposition 6. *Input–output linkages intensify persistence conditional on $J > 1$. Fix J, $\mathbf{\Gamma}$, and $\mathbf{\Lambda}$, and compare economies along the admissible scaling path $\mathbf{\Omega}(s) = s\,\bar{\mathbf{\Omega}}, \qquad s \in [\underline{s}, \bar{s}]$. Let $\mathbf{\Psi}(s) \equiv \mathbf{\Psi}^{NKOE}(s)$ denote the stable branch in $\boldsymbol{\mu}_t = \mathbf{\Psi}(s)\boldsymbol{\mu}_{t-1} + \cdots$. Consider the passive-policy limit ($\phi_\pi^n \to 0\ \forall n$). Suppose exactly N eigenvalues of $\mathbf{\Psi}(s)$ are zero, while the remaining $NJ - N$ eigenvalues are nonzero and stable.*[26] *Let $\lambda_*(s) \in (0,1)$ be the real, simple nonzero eigenvalue that governs the slowest decay: $|\lambda_*(s)| = \max\{|\lambda| : \lambda \in \sigma(\mathbf{\Psi}(s)),\ \lambda \neq 0\}$. Under the sufficient sign condition in Appendix D.7, $\lambda_*'(s) > 0$. Hence scaling up intermediate-input use raises the largest real non-zero eigenvalue of $\mathbf{\Psi}^{NKOE}$, which governs the slowest decay. Equivalently, conditional on $J > 1$, stronger input–output linkages make tariff-induced real marginal-cost deviations unwind more slowly.*

[26] We verify numerically that these assumptions hold.

Figure 4. Input-Output Linkages, Network Granularity and Persistence of Real Marginal Cost Deviations



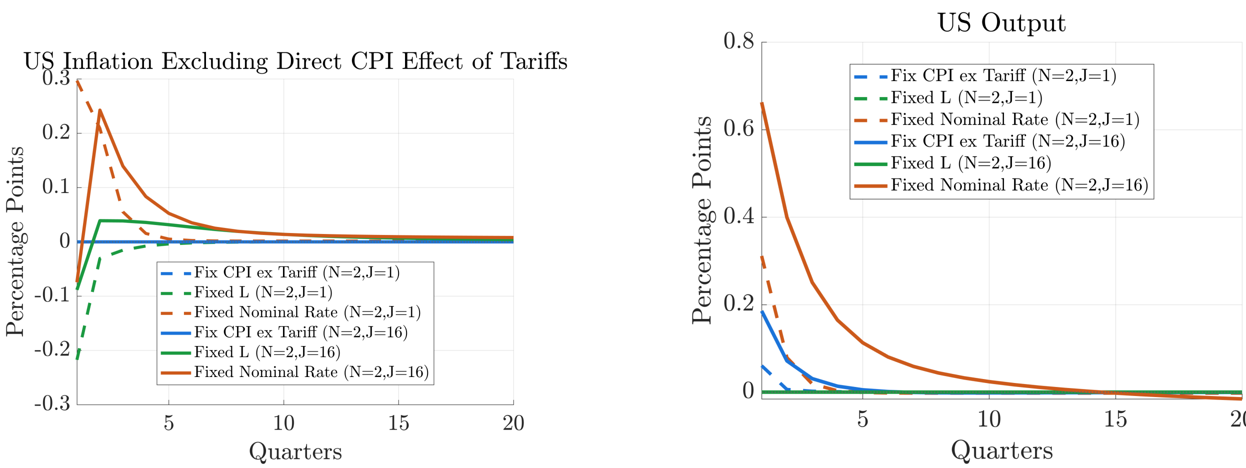
NOTE: The figure plots CPI-weighted real marginal cost in the tariff-imposing country. Panel (a) sets $J = 1$ and panel (b) sets $J = 16$. Within each panel, darker lines correspond to stronger input–output linkages, and the lightest line sets $\boldsymbol{\Omega} = \boldsymbol{0}$. The common y-axis shows that changing $\boldsymbol{\Omega}$ has little effect on persistence under $J = 1$ but a much larger effect under $J = 16$.

When $J > 1$, $\boldsymbol{\Psi}^{\text{NKOE}}$ solves $\mathcal{K}(0, \boldsymbol{\Psi}^{\text{NKOE}})\,\boldsymbol{\Psi}^{\text{NKOE}} = \boldsymbol{0}$, and, in the passive-policy limit along the scaling path ($\boldsymbol{\Omega}(s) = s\bar{\boldsymbol{\Omega}}$), $\boldsymbol{\Omega}$ enters only through $\boldsymbol{M}(s) \equiv (1+\beta)\boldsymbol{I} + (\boldsymbol{I} - s\bar{\boldsymbol{\Omega}})\boldsymbol{\Lambda} = (1+\beta)\boldsymbol{I} + \boldsymbol{\Lambda} - s\bar{\boldsymbol{\Omega}}\boldsymbol{\Lambda}$. Under the appendix conditions, a real simple nonzero eigenvalue $\lambda(s) \in (0, 1)$ of $\boldsymbol{\Psi}^{\text{NKOE}}(s)$ is linked to an eigenvalue $\zeta(s) = \beta\lambda(s) + \lambda(s)^{-1}$ of $\boldsymbol{M}(s)$. Scaling up s lowers this effective damping term and the stable-root relation then implies $\lambda'(s) > 0$. Applying this to the eigenvalue that governs the slowest decay gives Proposition 6: stronger input–output linkages make inherited real marginal-cost deviations unwind more slowly. See Appendix D.7 for the proof.

Figure 4 illustrates Proposition 6. Within each panel, darker lines correspond to stronger input–output linkages, lighter lines scale $\boldsymbol{\Omega}$ toward zero. Two margins stand out. First, moving from $J = 1$ to $J = 16$ substantially raises both the level and persistence of consumption-weighted real marginal cost deviations. Second, once $J > 1$, reducing $\boldsymbol{\Omega}$ compresses the real marginal cost response, and the effect is much larger when $J = 16$: in the $J = 1$ economy, shrinking $\boldsymbol{\Omega}$ mainly lowers the impact response while subsequent paths remain tightly clustered, whereas at $J = 16$ it lowers both the peak and the tail, showing that intermediate-input linkages materially slow the unwinding of tariff-induced cost distortions. Persistence is strongest when both forces operate jointly: $J > 1$ creates the endogenous propagation channel and $\boldsymbol{\Omega} \neq \boldsymbol{0}$ amplifies it.

Since the persistence of marginal cost deviations depends on network structure, inflation and output deviations can also be more persistent, as shown in Figure 5. Output is measured relative to the initial steady state rather than as a gap, so it need not converge to zero. Inflation typically returns to zero, but under policies stabilizing an aggregate quantity (e.g., consumption at steady state), adjustment may instead produce persistent inflation. We therefore define persistence as the number of periods required for variables to converge to their terminal values. Without portfolio adjustment costs, different policies generally yield different terminal steady states, so raw deviations from the initial steady state are not comparable across IRFs. Since our focus is *stabilization dynamics* rather than long-run incidence, the relevant comparison is the distance from each simulation's post-tariff equilibrium.

Figure 5. Network Granularity and Persistence of the Inflation-Output trade-off



NOTE: Unlike the rest of the paper, each series is reported relative to its case-specific terminal steady state, since different simulations converge to different long-run equilibria when portfolio adjustment costs are shut down. Under each policy regime, convergence is slower when $J > 1$: the multi-sector economy remains farther from its terminal equilibrium for longer than the $J = 1$ economy.

Figure 5 accordingly reports variables as deviations from their case-specific *terminal* steady states under three regimes: (i) passive policy, with $\rho^n_m = 1$ and $\phi^n_\pi \to 0$; (ii) perfect stabilization of aggregate CPI excluding tariffs at its pre-tariff level; and (iii) stabilization of aggregate employment. In all three cases, greater network granularity slows convergence: under $J > 1$, inherited cost distortions unwind more slowly, leaving the economy farther from its terminal allocation than under $J = 1$.

The theoretical results above build on and broaden the closed-economy network literature. Our model additionally allows us to address a distinct open-economy question: relative to treating the world as one closed economy, how does allowing $N > 1$ countries—each with its own nominal price level and monetary-policy rule, and linked through exchange rates—affect the persistence of real marginal-cost deviations?

Remark 4. In an open economy ($N > 1$), the threshold for systematic persistence of marginal cost deviations (i.e., $\boldsymbol{\Psi}^{\text{NKOE}} \neq \mathbf{0}$) remains $J > 1$.

Persistence arises from the mismatch between aggregate-demand dimension (N) and the dimension of rigidities generating lagged endogenous variables (NJ). Persistence therefore requires sectoral nominal rigidities; purely national rigidities, such as sticky wages alone, are insufficient. Although the open economy adds the exchange rate as a choice variable, country-level aggregate demand still cannot span all sectoral distortions, leaving the one-sector threshold unchanged.

The open economy instead changes the propagation matrix, not the threshold. In $\mathcal{K}(0, \boldsymbol{\Psi}^{\text{NKOE}})$, policy enters through the closed-economy labor-supply channel, $\boldsymbol{\alpha\Phi\Gamma\Lambda}$, and the policy-heterogeneity channel, $\boldsymbol{L}^P_{\mathcal{E}}\boldsymbol{Z\Phi\Gamma\Lambda}$. The latter operates only if $J > 1$, so inherited sectoral cost states exist; sectors use foreign intermediates, $\boldsymbol{L}^P_{\mathcal{E}} \neq \mathbf{0}$; and policy rules create a heterogeneous cross-country interest-rate response to those states, $\boldsymbol{Z\Phi\Gamma\Lambda} \neq \mathbf{0}$. This last condition can reflect different response coefficients, target baskets, or both.

To isolate the policy-heterogeneity channel, momentarily impose identical target baskets and let the Home–Foreign difference in policy reactiveness be $\delta = \phi^H_\pi - \phi^F_\pi$. As Appendix D.8 formalizes, perturbing the Home–Foreign policy difference around $\delta = 0$ can change the eigenvalue governing the slowest decay relative to the corresponding closed-economy block. The channel is UIP: different policy responses generate interest-rate differentials, which move exchange rates and hence imported-input costs. For example, after a Home tariff raises Home producer-price inflation, a more aggressive Home response appreciates the Home currency on impact and compresses imported-input costs, muting inflation today. Under appendix conditions, if the inherited pressure remains Home-inflationary and the exchange-rate-sensitive exposure is concentrated in Home sectors using Foreign intermediates, as this appreciation is expected to unwind, imported-input costs rise tomorrow. In that case, policy heterogeneity raises the persistence of real marginal-cost deviations relative to the closed-economy benchmark.

5 Quantitative Analysis

Our quantitative work is guided by two core questions: what is the macro impact of tariffs, and how does it change with production networks? We conduct two exercises. First, we feed the actual tariffs implemented from January 2025 to January 2026 as a shock sequence and show the model's predicted impact, which is stagflationary. Second, we study reversed tariff threats: agents observe a tariff path based on Liberation Day announcements and expect retaliation, yet no tariffs materialize at the implementation date. This highlights the role of expectations: even when tariffs are announced for only one period and never implemented, they leave behind distortions that take time to clear in global networks.

5.1 Data

We use the OECD Inter-Country Input-Output (ICIO) tables (Yamano et al., 2023) for 2022, aggregated to five countries (United States, Euro Area, China, Canada, Mexico) plus a rest-of-the-world block, and eight broad sectors: agriculture, energy, mining, food, basic manufacturing, advanced manufacturing, residential services, and other services, matched to the sectoral rigidity estimates of Nakamura and Steinsson (2008). As shown in Appendix F.1, services account for over 70% of U.S. GDP with most output consumed domestically, yet nearly one third of advanced-manufacturing inputs are imported, underscoring the importance of imported intermediates.

The empirical literature finds that tariff pass-through to consumer prices substantially exceeds exchange-rate pass-through, though estimates vary (e.g., Amiti et al., 2019; Fajgelbaum et al., 2020; Fajgelbaum and Khandelwal, 2022; Cavallo et al., 2021; Flaaen et al., 2019, 2025), and highlights the role of retail. Standard input-output tables net out retail and wholesale margins, attributing flows to originating sectors.[27] We therefore introduce a domestic importing sector that intermediates all imports, pays tariffs at the border, and faces sticky domestic prices, restoring the distribution margin absent from I-O tables.[28] We also introduce destination-specific pricing, indexing sector i by destination m so prices are sticky at the producer-importer pair level, and calibrate ϑ_{ni} to dollar-invoicing shares, setting it to zero for domestically sold goods.

Figure 6 shows the evolution of U.S. tariff rates from January 2025 to March 2026. We obtain country–sector tariffs from the WTO–IMF Tariff Tracker[29] at the HS 6-digit level and aggregate to ICIO sectors using 2024 bilateral import weights. Figure 6a shows that U.S. tariff rates reached 22.7% on May 3, 2025; as of March 2026, the effective rate implied by implemented measures is 10.8%. Figure 6b illustrates country-sector heterogeneity: rates reached as high as 160% on Chinese basic and advanced manufacturing goods.

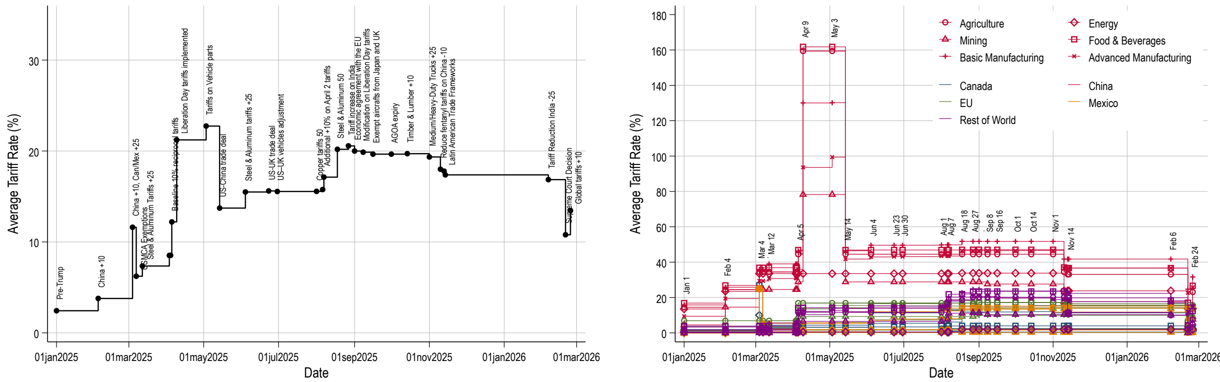
[27] See the U.S. Bureau of Economic Analysis's Concepts and Methods of the U.S. Input-Output Accounts for margin adjustments (Horowitz and Planting, 2009).

[28] Each original sector has a paired importing sector: in a given country, domestic steel producers and steel importers are separate sectors sharing the same stickiness parameter.

[29] `https://ttd.wto.org/en/analysis/tariff-actions`, last accessed March 31, 2026.

Figure 6. Since January 1, 2025

(a) Since January 1, 2025, (%) **(b)** Country-Sector Level Tariffs (%)



NOTE: Panel (a) reports effective U.S. tariff rates weighted by the trade volumes in 2024 and panel (b) country-sector-level effective U.S. tariff rates using the WTO–IMF Tariff Tracker. Rates are expressed in percent.

5.2 Calibration

Calibration parameters are summarized in Table 2. Sector-specific Calvo parameters follow Nakamura and Steinsson (2008), adjusted to quarterly frequency.[30] Following Atalay (2017), the elasticity between labor and the intermediate bundle is $\theta^X = 0.6$. For the elasticity across intermediate inputs, we follow Boehm et al. (2019) and Baqaee and Farhi (2024), setting $\theta_h^X = 0.2$. On the consumption side, we assume Cobb–Douglas preferences across sectors ($\theta_h^C = 1$), fixing sectoral expenditure shares. Boehm et al. (2023) estimate short-run trade elasticities of about 0.76 and long-run elasticities around 2; we conduct sensitivity analyses with θ_{li}^C and θ_{li}^X in [0.6, 2]. The baseline Armington elasticity across country varieties within a sector is $\theta_{li}^C = \theta_{li}^X = 1.5$, so goods are substitutes at the lowest level of aggregation.

For the Taylor rule, we assume: $1+i_{n,t} = (1+i_{n,t-1})^{\rho_m^n} (\Pi_{n,t})^{\phi_\pi^n} \quad \forall n \in N$. For the United States, we set $\rho_m^{\text{US}} = 1$ and take the limit $\phi_\pi^{\text{US}} \to 0$ to capture the Federal Reserve *looking through* tariff-related increases in CPI. For all other countries we set $\rho_m^n = 0.95$ following Clarida et al. (2000), with $\phi_\pi^{EA} = 1$ for the Euro Area, $\phi_\pi^{\text{MX}} = 0.3$ for Mexico, and $\phi_\pi^n = 0.2$ for the remaining blocks. These coefficients are calibrated using $\phi_\pi^n = (1-\rho_m^n)/\overline{\pi}_n^C$, where $\overline{\pi}_n^C$ is the long-run average of quarterly CPI inflation in country n, computed from 2002Q2 to 2024Q4 data.

The analytical model assumes AR(1) tariffs. For the quantitative exercises, we instead

[30] We map Calvo updating frequencies to Rotemberg adjusting costs. Additionally, we conduct robustness exercises with the updating frequencies of Pasten et al. (2024). For less granular variants (e.g., $J = 1$), we use a weighted average of the diagonal elements of $\mathbf{\Lambda}$ rather than average updating frequencies.

feed a shock that raises tariffs for 100 periods (25 years), which over our horizon produces results similar to $\rho_\tau = 0.999$. We solve the nonlinear model in Dynare under perfect foresight with MIT shocks.

The model incorporates a permanent real capital account wedge in each country so that 2018 data can serve as the steady state. The United States maintains persistent trade deficits alongside negative net foreign assets, which is inconsistent with standard steady-state algebra; the wedges reconcile this by allowing trade deficits and net debt to coexist in steady state. They can be interpreted as persistent cross-country differences in patience or, equivalently, as an exogenous spread between interest paid on assets and liabilities.

Table 2. Parameter values

Parameter	Explanation	Value	Source
σ	Intertemporal elasticity of substitution (EoS)	2	Standard
η	Elasticity of Labor	1	Standard
ψ	Portfolio adjustment cost	0.001	Standard
ρ_m^n	Inertia in Taylor Rule for $n \neq US$	0.95	Clarida et al. (2000)
ρ_m^{US}	Inertia in Taylor Rule for U.S.	0.82-1	Carvalho et al. (2021)
ϕ_π^{US}	Weight on inflation in Taylor Rule for U.S.	0-1.29	Carvalho et al. (2021)
λ_n	Sector specific price rigidities		Nakamura and Steinsson (2008)
θ^X	EoS between intermediates and VA	0.6	Atalay (2017)
θ_h^C	Intratemporal EoS of consumption among sectors	1	di Giovanni et al. (2023)
θ_h^X	EoS among intermediate inputs	0.2	Baqaee and Farhi (2019); Boehm et al. (2019)
θ_{li}^C	Sector level consumption bundle EoS	0.6-2	di Giovanni et al. (2023)
θ_{li}^X	Sector level input bundle EoS	0.6-2	di Giovanni et al. (2023)

As a validation exercise, we study the 2018 trade war episode, focusing on U.S. tariffs imposed on China and other trading partners between February and September 2018. In this period, the U.S. implemented tariffs ranging from 10% to 25% on China, a 10% tariff on aluminum, 25% on iron and steel, 30% on solar panels, and 20 to 50% tariffs on washers, with some exceptions at the country level. In return, Canada, China, the European Union, Mexico, Russia, and Turkey retaliated with tariffs ranging from 5 to 20%. We obtained the detailed tariff data for this episode from Fajgelbaum et al. (2020) and trade values from the USITC website to calculate the weighted tariff rates.[31] For this episode, our model predicts a 3% nominal appreciation of the U.S. dollar (USD) against the Chinese yuan on impact, eventually reaching a 4.2% nominal appreciation in the long run. This aligns with the observed 5.6% appreciation of the USD between June 2018 and December 2018. Real GDP loss reaches 0.1 percentage points. This is within the range of the estimate in Fajgelbaum et al. (2020), which found an aggregate real income loss of 0.04% of GDP. Finally, the model predicts an inflation impact of 0.27 percentage points, which is close to the 0.1–0.2 percentage

[31] Exports: https://dataweb.usitc.gov/trade/search/TotExp/HTS, Imports: https://dataweb.usitc.gov/trade/search/GenImp/HTS.

point estimate in Barbiero and Stein (2025).

5.3 Implemented Tariffs

Next, we feed in implemented tariffs in each period, assuming they persist for 100 periods, and overlay the resulting IRFs. In the baseline model, tariffs generate dollar appreciation in real and nominal terms for reasonable parameters, because $\hat{w}_t$ is sufficiently negative. The observed post–Liberation Day depreciation therefore requires an additional force that renders the wedge sufficiently positive. We capture that force with a reduced-form UIP premium κ_t, motivated by the empirically observed rise in UIP deviations in this period (Kalemli-Özcan et al., 2026). This modeling choice is consistent with several underlying mechanisms, including tariff-induced uncertainty or a decline in the dollar convenience yield.[32] Accordingly, the UIP condition becomes $\frac{1+i_{n,t}}{1+i_t^{US}} = E_t\left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}}\right]\frac{1+\kappa_t}{1-\psi'(B_{n,t}^{US}/P_t^{US})}$. Kalemli-Özcan et al. (2026) find that the UIP premium rose by 2.98 percentage points in 1Q2025 relative to 4Q2024, remaining elevated at 1.62 and 0.52 percentage points over the next two quarters.[33] We set $\kappa_0 = 0.0298$, 0.0162, and 0.0052 for tariff changes introduced in the first, second, and third quarters, respectively.

As shown in Figure 7, implemented tariffs result in a 0.15% U.S. real GDP expansion on impact that reverses to a 0.4% decline in subsequent periods. Consumption falls by 0.03%, net exports rise by 0.05 percentage points of steady-state GDP, inflation increases by 0.13 percentage points, and the trade-weighted NEER depreciates by 1.90%.

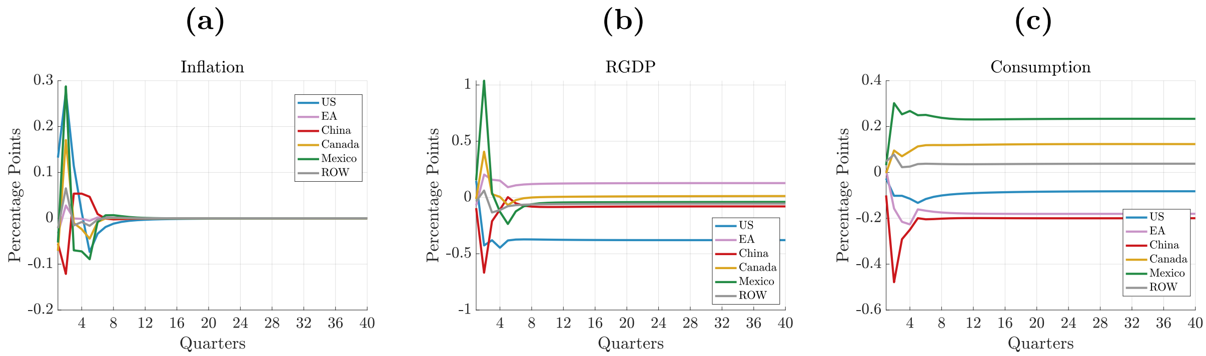
Effects are heterogeneous across trading partners. China experiences the largest contraction: real GDP falls by 0.09%, consumption by 0.10%, real wages by 0.29%, and net exports by 0.25 percentage points of steady-state GDP, while inflation declines by 0.06 percentage points. Mexico benefits the most, with real GDP rising by 0.15%, consumption by 0.03%, real wages by 0.22%, and net exports by 0.21 percentage points of steady-state GDP; inflation falls by 0.05 percentage points. Canada is nearly unchanged in aggregate activity (real GDP −0.01%) but its external balance improves, with net exports rising by 0.20 percentage points of steady-state GDP; inflation falls by 0.07 percentage points. The Euro Area contracts modestly (real GDP −0.07%, inflation −0.03 pp), while consumption rises slightly by 0.01%. The rest of the world also contracts (real GDP −0.03%, inflation −0.02 pp), though

[32] We view this reduced-form UIP premium as an empirically-disciplined shorthand: even in a richer asset-market environment that endogenizes the premium, tariff-induced depreciation would still operate at least in part through a UIP wedge to counteract appreciationary forces. This is because the UIP premium interacts with the risk-sharing wedge from Section 3, which in this setup becomes $\hat{w}_t + \kappa_t = \hat{Q}_t - \sigma(\hat{C}_{H,t} - \hat{C}_{F,t})$. Even when tariffs generate $\hat{w}_t < 0$, a sufficiently large κ_t can overcome the appreciationary pressure, yielding depreciation and lower relative consumption.

[33] As in Kalemli-Özcan et al. (2026), we shift quarters by 20 days so that 1Q2025 begins on Inauguration Day.

consumption rises by 0.05%. Notably, the tariff shock is inflationary on impact only for the United States; all non-U.S. regions experience deflation, reflecting that the implemented tariffs raise U.S. domestic prices directly but act primarily as a negative external demand shock abroad.

Figure 7. Implemented Tariffs



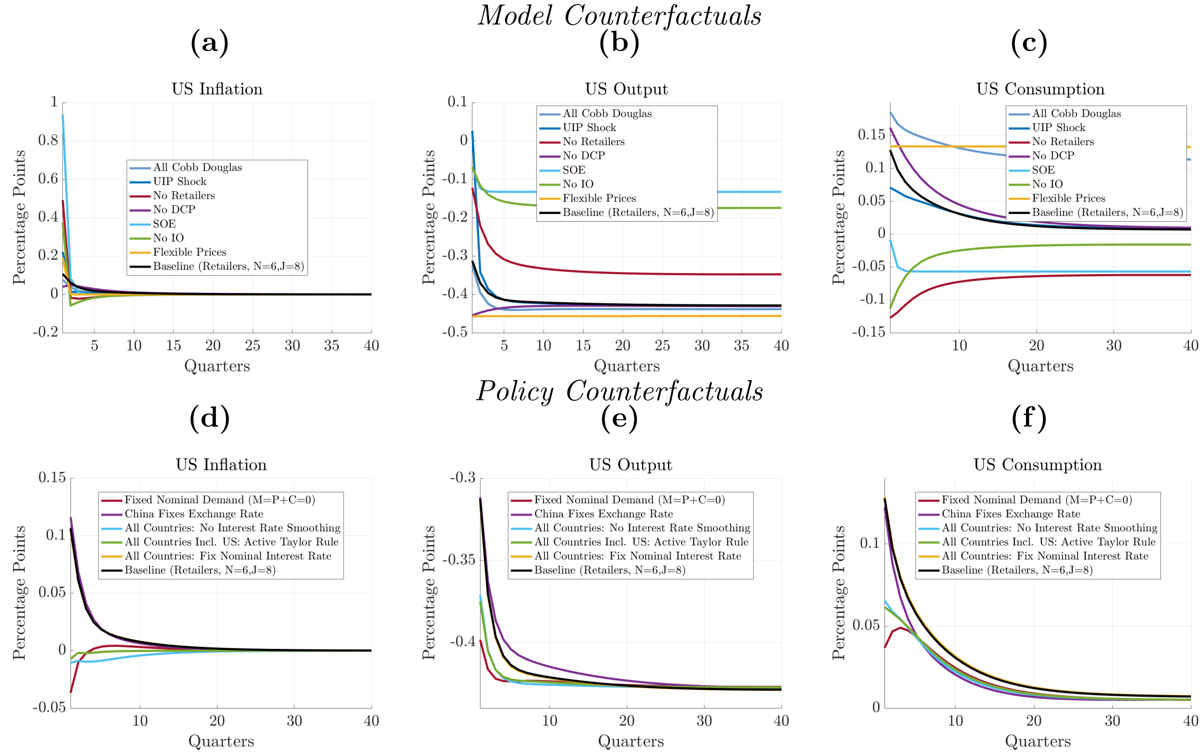
NOTE: Figure 7 visualizes simulated responses to the 2025-2026 U.S. tariff packages, targeting China, Canada, Mexico, Europe and the RoW. Impulse responses are computed with MIT shocks. Instead of a single MIT shock, the figure overlays impulse responses to successive tariff changes, treating each as a new tariff level, and adds a one-time UIP premium shock at the episode's start.

5.4 Model and Policy Counterfactuals

We next explore the sensitivity of our baseline results to alternative model features and policy specifications, clarifying which ingredients and policy responses matter most quantitatively. Figure 8, top row, depicts model counterfactuals. "Baseline" introduces cumulative tariffs implemented as of 1Q2026 from Section 5.3. "All Cobb–Douglas" sets all elasticities to $\theta = 1$. "UIP Shock" adds a one-time shock to the UIP equation in the spirit of Kalemli-Özcan et al. (2026). "No Retailers" removes the domestic importing sector. "No DCP" sets $\vartheta_{ni} = 0 \; \forall n, i$, collapsing to PCP. "SOE" makes the rest of the world arbitrarily large, fixing foreign variables at steady state, with $\mathbf{\Omega} = \mathbf{0}$ and no DCP. "No IO" sets $\mathbf{\Omega} = \mathbf{0}$ (retailers are also absent when $\mathbf{\Omega} = \mathbf{0}$). "Flexible Prices" replaces the NKPC with $MC_{ni,t} = P_{ni,t}$ in every sector. Removing retailers, as in "SOE" and "No Retailers," implies full and more immediate passthrough to consumer prices; beyond this, inflation impulses are similar across specifications. Output responses are close after differing on impact. For consumption, each scenario generates a slightly different risk-sharing wedge through differing terms-of-trade dynamics, creating level differences. Notably, scenarios with more immediate passthrough, such as "No Retailers," generate a negative consumption response.

Figure 8, bottom row, reports policy counterfactuals relative to the same baseline, varying

Figure 8. Model and Policy Counterfactuals



NOTE: Top row: model counterfactuals. Bottom row: policy counterfactuals. Panels plot U.S. inflation, real GDP, and consumption relative to the baseline cumulative tariff path implemented as of 1Q2026 from Section 5.3. Each line changes one model feature or one policy rule at a time, holding the remaining environment fixed.

the policy rule while holding the tariff path fixed. "Fixed Nominal Demand" imposes $M_t = P_t + C_t = 0$, fixing nominal spending at its steady-state level. "China Fixes Exchange Rate" stabilizes China's bilateral exchange rate. "All Countries: No Interest Rate Smoothing" removes policy inertia in all countries. "All Countries Incl. US: Active Taylor Rule" makes the Fed target CPI with positive weight (setting $\rho_m^{\text{US}} = 0.82$ and $\phi_\pi^{\text{US}} = 1.29$ following Carvalho et al. (2021)), as other countries do in the baseline, instead of looking through. "All Countries: Fix Nominal Interest Rate" holds policy rates fixed everywhere. Output and consumption responses are similar across specifications. Cases that make the central bank reactive to CPI, such as fixed nominal demand, can turn U.S. inflation slightly negative on impact.

5.5 Reversed Tariff Threats

Motivated by unimplemented tariffs observed in practice, we simulate a reversed tariff threat: in period 1, the United States announces tariffs for period 2, and agents expect symmetric retaliation by other countries for the same period. When period 2 arrives, no tariffs are levied by either side, isolating the macroeconomic effects of credible announcements operating purely through expectations.

We construct two impulse responses under perfect foresight. The first simulates the actual tariff shock with retaliation, announced and implemented in period 1. The second simulates the same shock announced to take effect in period 2, only to be withdrawn before implementation. The reversed-threat impulse response is obtained by shifting the first response forward one period and subtracting it from the second, isolating the pure expectations effect. This approach is inspired by the *fake news* algorithm of Auclert et al. (2021), who use reversals of shocks as a computational device for sequence-space solutions; we apply it to study macroeconomic implications of trade policy reversals.

Figure 9 compares the reversed-threat scenario with actual tariffs under retaliation. On impact, U.S. inflation rises by 0.34 percentage points, consumption falls by 0.25%, real GDP rises by 0.27%, and the NEER depreciates by 2.66%. Once the reversal is revealed in period 2, the NEER appreciates immediately given its forward-looking nature, while inflation, consumption, and output take 4-12 quarters to return to steady-state levels.

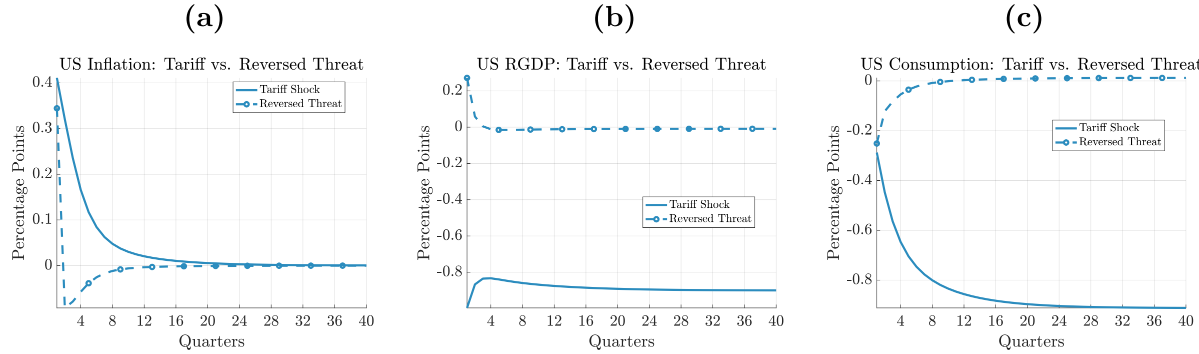
The USD depreciation is driven by expected retaliation. Because the Liberation Day tariffs are large and the U.S. is smaller than the rest of the world, symmetric retaliation is sufficient to make the risk-sharing wedge positive, which, as shown in Section 3, lowers consumption through the intertemporal wealth effect. Once the shock is announced, $\hat{w}_t$ jumps immediately and the exchange rate adjusts even though tariffs take effect only next period. Agents lower consumption before tariffs are implemented, so the on-impact effect of the reversed threat is comparable to the actual tariff. With a forward-looking NKPC, inflation is also similar across the two scenarios.[34] Real GDP rises modestly on impact, as agents perceive the pre-tariff period as less distortionary for production and depreciation raises U.S. export competitiveness. This exercise confirms that the expectations channel alone generates sizable macroeconomic effects, and that the distortions it introduces do not dissipate immediately upon reversal, even when no tariffs are actually implemented.

6 Conclusion

This paper develops a multi-country, multi-sector New Keynesian open-economy framework to study the macroeconomic impact of trade distortions when global production networks,

[34] When retailers are absent and households pay tariffs directly, the inflation responses diverge, because the reversed-threat scenario lacks the direct passthrough to consumer prices that the actual-tariff scenario produces.

Figure 9. Impact of Reversed Tariff Threats



NOTE: Figure 9 visualizes simulated responses to reversed tariff announcements. Tariffs are announced in the first period, with retaliation expected, and later canceled in the second period.

heterogeneous sectoral price rigidities, and incomplete financial markets operate jointly. Our central message is that the open-economy and network dimensions are not separable refinements of canonical NKOE models: they interact in ways that alter both the sign and persistence of tariff effects, and they do so through two objects absent from existing frameworks.

The first object is the risk-sharing wedge. Under incomplete markets, tariffs open a martingale wedge summarizing the wealth transfer across countries in global general equilibrium. Its sign, shaped jointly by the production network structure (through terms of trade) and by shock persistence (through intertemporal substitution), determines whether the tariff-imposing country gains or loses. Neither complete-markets nor static exogenous-transfer closures are innocuous: the former shuts down the wealth-transfer channel by construction, the latter overstates it. A tractable incomplete-markets structure is thus an essential ingredient of tariff analysis, not a technical detail.

The second object is the NKOE propagation matrix. With multiple sectors linked through input-output networks, tariffs generate real marginal cost distortions that become inherited states: the dimensionality of sectoral rigidities exceeds that of country-level aggregate demand, so monetary policy cannot span all sectoral gaps, even when the exchange rate is a separate national instrument. Transitory tariff shocks thus leave persistent distortions, which intermediate-input linkages amplify. This is a distinctly open-economy network result: exchange-rate adjustment does not eliminate propagation, and the one-sector NKOE benchmark, in which inflationary impulses are exhausted on impact, understates both the inflation–output trade-off and the stabilization burden on monetary policy.

Applied to the 2025–2026 episode, the framework delivers three quantitative lessons. Implemented U.S. tariffs are stagflationary, with sizable and heterogeneous international spillovers. Open-economy models without input-output linkages overstate the inflationary impact and understate the output decline, missing slow-moving propagation across sectors, countries, and time. And reversed tariff threats generate persistent macroeconomic effects through the expectations channel.

Our finding that sectoral distortions exceed what N-country monetary policy can offset invites normative analysis of international policy coordination in environments where the production network, rather than aggregate shocks, generates the residual instability. We leave this to future work.

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Supplemental Appendix

A Approximated Linear Equilibrium Conditions

The linearized equilibrium conditions used to arrive at the five-equation Global New Keynesian representation are:

$$E_t\hat{C}_{n,t+1} - \hat{C}_{n,t} = \frac{1}{\sigma}\left(\hat{i}_t - E_t\pi_{n,t+1}\right) \tag{A.1}$$

$$\hat{i}_{n,t} - \hat{i}_{US,t} = E_t\hat{\mathcal{E}}_{n,t+1} - \hat{\mathcal{E}}_{n,t} + \hat{\psi} \tag{A.2}$$

$$\hat{\mathcal{E}}_{n,m,t} = \hat{\mathcal{E}}^{US}_{n,t} - \hat{\mathcal{E}}^{US}_{m,t} \tag{A.3}$$

$$\hat{\mathcal{E}}_{n,n,t} = 0 \tag{A.4}$$

$$\hat{W}_{n,t} - \hat{P}^C_{n,t} = \eta\hat{L}_{n,t} + \sigma\hat{C}_{n,t} \tag{A.5}$$

$$\hat{C}_{nt} = \sum_{j\in J}\Gamma_{n,j}\hat{C}_{n,j,t} \tag{A.6}$$

$$\hat{C}_{n,j,t} = \sum_{m\in N}\Gamma_{n,j,mj}\hat{C}_{n,mj,t} \tag{A.7}$$

$$\hat{P}_{n,mj,t} = \hat{\mathcal{E}}_{n,m,t} + \hat{\tau}_{n,m,t} + \hat{P}_{mj,t} \tag{A.8}$$

$$\hat{C}_{n,j,t} = \hat{C}_{n,t} - \theta^C_h\left(\hat{P}^C_{n,j,t} - \hat{P}^C_{n,t}\right) \tag{A.9}$$

$$\hat{C}_{n,mj,t} = \hat{C}_{n,j,t} - \theta^C_{l,j}\left(\hat{P}^C_{n,mj,t} - \hat{P}^C_{n,j,t}\right) \tag{A.10}$$

$$\hat{X}_{ni,j,t} = \sum_{m\in N}\Omega_{ni,j,mj}\hat{X}_{ni,mj,t} \tag{A.11}$$

$$\hat{X}_{ni,mj,t} = \hat{X}_{ni,j,t} - \theta^X_{l,j}\left(\hat{P}_{n,mj,t} - \hat{P}^X_{ni,j,t}\right) \tag{A.12}$$

$$\hat{X}_{ni,t} = \sum_{j\in J}\Omega_{ni,j}\hat{X}_{ni,j,t} \tag{A.13}$$

$$\hat{X}_{ni,j,t} = \hat{X}_{ni,t} - \theta^X_h\left(\hat{P}^X_{ni,j,t} - \hat{P}^X_{ni,t}\right) \tag{A.14}$$

$$\hat{Y}_{ni,t} = \hat{A}_{ni,t} + \alpha_{ni}\hat{L}_{ni,t} + (1-\alpha_{ni})\hat{X}_{ni,t} \tag{A.15}$$

$$\widehat{MC}_{ni,t} = -\hat{A}_{ni,t} + \alpha_{ni}\hat{W}_{n,t} + (1-\alpha_{ni})\hat{P}^X_{ni,t} \tag{A.16}$$

$$\hat{X}_{ni,t} - \hat{L}_{ni,t} = \theta^X\hat{W}_{n,t} - \theta^X\hat{P}^X_{ni,t} \tag{A.17}$$

$$\pi_{ni,t} = \frac{\theta^R}{\delta_{ni}}\left(\widehat{MC}_{ni,t} - \hat{P}_{ni,t}\right) + \beta\mathbb{E}_t\pi_{ni,t+1} \tag{A.18}$$

$$\bar{B}^{US}\hat{B}^{US}_t = \sum_{m}^{N-1}\bar{B}^{US}_m\hat{B}^{US}_{m,t} \tag{A.19}$$

$$\bar{Y}_{ni}\hat{Y}_{ni,t} = \sum_{n\in\mathcal{N}} \bar{C}_{m,ni}\hat{C}_{m,ni,t} + \sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{X}_{mj,ni}\hat{X}_{mj,ni,t}, \tag{A.20}$$

$$\bar{L}_n\hat{L}_{n,t} = \sum_{i\in J} \bar{L}_{ni}\hat{L}_{ni,t} \tag{A.21}$$

$$\pi_{n,t} = \hat{P}^C_{n,t} - \hat{P}^C_{n,t-1} \tag{A.22}$$

$$\hat{i}_{n,t} = \phi_\pi \pi_{n,t} + \hat{M}_{n,t}. \tag{A.23}$$

The linearized external budget constraint is

$$\begin{aligned}
&\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{C}_{n,mj}(\hat{P}_{n,mj,t} + \hat{C}_{n,mj,t}) + \sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{X}_{ni,mj}(\hat{P}_{n,mj,t} + \hat{X}_{ni,mj,t}) \\
&\quad + \bar{\mathcal{E}}_n(1+\bar{i}^{US}_n)\bar{B}^{US}_n\left(\hat{\mathcal{E}}_{n,t} + \hat{i}^{US}_{n,t-1} + \hat{B}^{US}_{n,t-1}\right) \\
&= \sum_i \bar{P}_{ni}\bar{Y}_{ni}(\hat{P}_{ni,t} + \hat{Y}_{ni,t}) + \bar{\mathcal{E}}_n\bar{B}^{US}_n(\hat{\mathcal{E}}_{n,t} + \hat{B}^{US}_{n,t}),
\end{aligned} \tag{A.24}$$

where bars denote steady-state values.

B Relating the Balance of Payments to Prices

Define gross dollar debt by $V^{US}_{n,t} \equiv (1+i^{US}_t)B^{US}_{n,t}$. Using $\bar{\mathcal{E}}_n = 1$, $1+\bar{i}^{US} = \beta^{-1}$, and $\overline{NX}_n = (1-\beta)\bar{V}^{US}_n$, the linearized balance-of-payments condition implies

$$\beta\hat{V}^{US}_{n,t} - \hat{V}^{US}_{n,t-1} = (1-\beta)\hat{\mathcal{E}}_{n,t} - (1-\beta)\widehat{NX}_{n,t} + \beta\hat{i}^{US}_t. \tag{B.1}$$

Stacking across countries and replacing one debt equation with U.S.-bond market clearing yields $\beta\hat{\boldsymbol{V}}_t = \boldsymbol{\Xi}_1\hat{\boldsymbol{V}}_{t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}^P_t + \boldsymbol{\Xi}_4\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{\Xi}_5\hat{\boldsymbol{\tau}}_t$. To arrive at this expression, we express net exports in terms of $(\hat{\boldsymbol{C}}_t, \hat{\boldsymbol{P}}^P_t, \hat{\boldsymbol{\mathcal{E}}}_t, \hat{\boldsymbol{\tau}}_t)$. Their baseline decomposition is

$$\underbrace{\overline{\boldsymbol{NX}}_n}_{N\times N}\underbrace{\widehat{\boldsymbol{NX}}_{n,t}}_{N\times 1} = \underbrace{\overline{\boldsymbol{Y}}^{N,NJ}}_{N\times NJ}\left[(\hat{\boldsymbol{P}}^P_t + \hat{\boldsymbol{Y}}_{ni,t}) - \boldsymbol{\alpha}\left(\hat{\boldsymbol{P}}^{C,\tau}_t + \hat{\boldsymbol{C}}_t\right) - \boldsymbol{\Omega}\left(\hat{\boldsymbol{P}}^{X,\tau}_{ni,t} + \hat{\boldsymbol{X}}_{ni,t}\right)\right]. \tag{B.2}$$

When an object that is inherently lower-dimensional is mapped to a higher dimension, we replicate rows or columns to reach the final object that has the right dimensions. For example, steady-state outputs are $NJ \times 1$, but the new object is $N \times NJ$: we transpose the vector and repeat it N times to arrive at $\overline{\boldsymbol{Y}}^{N,NJ}$.

B.1 Market-Clearing Condition

Linearized goods-market clearing is

$$\overline{\boldsymbol{Y}}^{ni}\hat{\boldsymbol{Y}}_{ni,t} = \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\hat{\boldsymbol{C}}_t^{nmj} + \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\hat{\boldsymbol{X}}_t^{nimj},$$

where $\boldsymbol{\Gamma}^M$ is $NJ \times N^2J$ and $\boldsymbol{\Omega}^M$ is an $NJ \times N^2J^2$ matrix, obtained by shifting $\boldsymbol{\Gamma}$ and $\boldsymbol{\Omega}$ so that multiplication loads onto the correct entries. Relative-demand conditions imply[35]

$$\begin{aligned}\hat{\boldsymbol{C}}_t^{nmj} &= \boldsymbol{S}_1\hat{\boldsymbol{C}}_t + \theta^C\left(\boldsymbol{S}_1\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{nmj}\right),\\ \hat{\boldsymbol{X}}_t^{nimj} &= \boldsymbol{S}_2\hat{\boldsymbol{X}}_{ni,t} + \theta^X\left(\boldsymbol{S}_2\hat{\boldsymbol{P}}_{ni,t}^P - \hat{\boldsymbol{P}}_{ni,mj,t}^P\right),\end{aligned}$$

where the $\boldsymbol{S}$ matrices are selector matrices that pick the appropriate producer price, bilateral exchange rate, and tariff entries, and

$$\hat{\boldsymbol{P}}_t^{nmj} = \boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{S}_5\hat{\boldsymbol{\tau}}_t, \qquad \hat{\boldsymbol{P}}_{ni,mj,t}^P = \boldsymbol{S}_6\hat{\boldsymbol{P}}_t^{nmj}.$$

Using the selector identities

$$\overline{\boldsymbol{C}}_n\,\boldsymbol{\Gamma}^M\boldsymbol{S}_1 = \boldsymbol{\Gamma}^\top\,\overline{\boldsymbol{C}}, \qquad \boldsymbol{\Omega}^M\boldsymbol{S}_2 = \boldsymbol{\Omega}^\top, \qquad \boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_3 = \boldsymbol{\Omega}^\top,$$

and

$$\boldsymbol{L}_{\mathcal{E}}^P \equiv \boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_4, \qquad \boldsymbol{L}_{\tau}^P \equiv \boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_5,$$

we obtain

$$\begin{aligned}\hat{\boldsymbol{Y}}_{ni,t} = {} & \overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\hat{\boldsymbol{C}}_t + \boldsymbol{\Omega}^\top\hat{\boldsymbol{X}}_{ni,t}\\ & + \theta^C\overline{\boldsymbol{Y}}^{ni^{-1}}\left(\boldsymbol{T}_P^C\hat{\boldsymbol{P}}_t^P + \boldsymbol{T}_{\mathcal{E}}^C\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{T}_\tau^C\hat{\boldsymbol{\tau}}_t\right)\\ & + \theta^X\left(\left[\boldsymbol{\Omega}^\top\boldsymbol{\Omega} - \boldsymbol{\Omega}^\top\right]\hat{\boldsymbol{P}}_t^P + \left[\boldsymbol{\Omega}^\top\boldsymbol{L}_{\mathcal{E}}^P - \boldsymbol{L}_{\mathcal{E}}^P\right]\hat{\boldsymbol{\mathcal{E}}}_t + \left[\boldsymbol{\Omega}^\top\boldsymbol{L}_\tau^P - \boldsymbol{L}_\tau^P\right]\hat{\boldsymbol{\tau}}_t\right),\end{aligned} \tag{B.3}$$

where

$$\boldsymbol{T}_P^C \equiv \boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\boldsymbol{\Gamma} - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_3, \quad \boldsymbol{T}_{\mathcal{E}}^C \equiv \boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\boldsymbol{L}_{\mathcal{E}}^C - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_4,$$

$$\boldsymbol{T}_\tau^C \equiv \boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}}\boldsymbol{L}_\tau^C - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_5.$$

[35] We abstract from higher- and lower-level θ varieties; the results become more complicated with additional elasticities but the intuition is unchanged. Mathematically, we take $\theta_h^X = \theta_{l,j}^X = \theta^X \quad \forall j \in \mathcal{J}$ and $\theta_h^C = \theta_{l,j}^C = \theta^C \quad \forall j \in \mathcal{J}$.

B.2 Substituting out $\hat{X}_{ni,t}$

From the production bundle and production function,

$$\hat{X}_{ni,t} = \hat{Y}_{ni,t} + \theta^X \alpha_{ni}(\hat{W}_t - \hat{P}^X_{ni,t}),$$

so in vector form

$$\hat{\boldsymbol{X}}_{ni,t} = \hat{\boldsymbol{Y}}_{ni,t} + \theta^X(\boldsymbol{\alpha}\hat{\boldsymbol{W}}_t - \tilde{\boldsymbol{\alpha}}\hat{\boldsymbol{P}}^P_{ni,t}).$$

Here, $\boldsymbol{\alpha}$ is $NJ \times N$ matrix with entries $\boldsymbol{\alpha}_{nj,n} = \alpha_{nj}$ and all other entries 0. $\tilde{\alpha}$, on the other hand, is the diagonal matrix whose diagonal entries are α_{nj}. Substituting this into (B.3), and then using

$$\hat{\boldsymbol{W}}_t = \hat{\boldsymbol{P}}^C_t + \sigma\hat{\boldsymbol{C}}_t, \qquad \hat{\boldsymbol{P}}^C_t = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^C_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}^C_{\tau}\hat{\boldsymbol{\tau}}_t,$$

$$\hat{\boldsymbol{P}}^X_{ni,t} = \boldsymbol{\Omega}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^P_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}^P_{\tau}\hat{\boldsymbol{\tau}}_t, \qquad \boldsymbol{\Psi}_T \equiv (\boldsymbol{I} - \boldsymbol{\Omega}^\top)^{-1},$$

gives

$$\begin{aligned}\hat{\boldsymbol{Y}}_{ni,t} = \boldsymbol{\Psi}_T \Bigg[&\Big(\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{\Gamma}^\top\overline{\boldsymbol{C}} + \theta^X\boldsymbol{\Omega}^\top\boldsymbol{\alpha}\sigma\Big)\hat{\boldsymbol{C}}_t \\ &+ \Big(\theta^C\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{T}^C_P + \theta^X\boldsymbol{\Omega}^\top\boldsymbol{T}^P_P\Big)\hat{\boldsymbol{P}}^P_t \\ &+ \Big(\theta^C\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{T}^C_{\mathcal{E}} + \theta^X\boldsymbol{\Omega}^\top\boldsymbol{T}^X_{\mathcal{E}}\Big)\hat{\boldsymbol{\mathcal{E}}}_t \\ &+ \Big(\theta^C\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{T}^C_{\tau} + \theta^X\boldsymbol{\Omega}^\top\boldsymbol{T}^P_{\tau}\Big)\hat{\boldsymbol{\tau}}_t\Bigg],\end{aligned} \tag{B.4}$$

with

$$\boldsymbol{T}^P_P \equiv \boldsymbol{\Omega} - \boldsymbol{I} + \boldsymbol{\alpha}\boldsymbol{\Gamma} - \tilde{\boldsymbol{\alpha}}\boldsymbol{\Omega},$$

$$\boldsymbol{T}^X_{\mathcal{E}} \equiv \boldsymbol{L}^X_{\mathcal{E}} - (\boldsymbol{\Omega}^\top)^{-1}\boldsymbol{L}^X_{\mathcal{E}} + \boldsymbol{\alpha}\boldsymbol{L}^C_{\mathcal{E}} - \tilde{\boldsymbol{\alpha}}\boldsymbol{L}^X_{\mathcal{E}}, \qquad \boldsymbol{T}^P_{\tau} \equiv \boldsymbol{L}^P_{\tau} - (\boldsymbol{\Omega}^\top)^{-1}\boldsymbol{L}^P_{\tau} + \boldsymbol{\alpha}\boldsymbol{L}^C_{\tau} - \tilde{\boldsymbol{\alpha}}\boldsymbol{L}^P_{\tau}.$$

Substituting (B.4) into (B.2) gives

$$\overline{\boldsymbol{NX}}_n\widehat{\boldsymbol{NX}}_{n,t} = \Xi_P\,\hat{\boldsymbol{P}}^P_t + \Xi_{\mathcal{E}}\,\hat{\boldsymbol{\mathcal{E}}}_t + \Xi_\tau\,\hat{\boldsymbol{\tau}}_t + \Xi_C\,\hat{\boldsymbol{C}}_t, \tag{B.5}$$

where $\boldsymbol{\Psi}_\Delta \equiv (\boldsymbol{I} - \boldsymbol{\Omega})(\boldsymbol{I} - \boldsymbol{\Omega}^\top)^{-1} = (\boldsymbol{I} - \boldsymbol{\Omega})\boldsymbol{\Psi}^\top$ and

$$\Xi_P = \overline{\boldsymbol{Y}}^{NNJ}\Big\{\boldsymbol{\Psi}_\Delta\Big(\theta^C\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{T}^C_P + \theta^X\boldsymbol{\Omega}^\top\boldsymbol{T}^P_P\Big) - \theta^X\boldsymbol{\Omega}\,(\boldsymbol{\alpha}\boldsymbol{\Gamma} - \tilde{\boldsymbol{\alpha}}\boldsymbol{\Omega}) + \Big[\boldsymbol{I} - \boldsymbol{\alpha}\boldsymbol{\Gamma} - \boldsymbol{\Omega}^2\Big]\Big\},$$

$$\boldsymbol{\Xi}_{\mathcal{E}} = \overline{\boldsymbol{Y}}^{NNJ}\left\{\boldsymbol{\Psi}_{\Delta}\left(\theta^{C}\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{T}_{\mathcal{E}}^{C} + \theta^{X}\boldsymbol{\Omega}^{\top}\boldsymbol{T}_{\mathcal{E}}^{X}\right) - \theta^{X}\boldsymbol{\Omega}\left(\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^{C} - \tilde{\boldsymbol{\alpha}}\boldsymbol{L}_{\mathcal{E}}^{P}\right) - \left[\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^{C} + \boldsymbol{\Omega}\boldsymbol{L}_{\mathcal{E}}^{P}\right]\right\},$$

$$\boldsymbol{\Xi}_{\tau} = \overline{\boldsymbol{Y}}^{NNJ}\left\{\boldsymbol{\Psi}_{\Delta}\left(\theta^{C}\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{T}_{\tau}^{C} + \theta^{X}\boldsymbol{\Omega}^{\top}\boldsymbol{T}_{\tau}^{P}\right) - \theta^{X}\boldsymbol{\Omega}\left(\boldsymbol{\alpha}\boldsymbol{L}_{\tau}^{C} - \tilde{\boldsymbol{\alpha}}\boldsymbol{L}_{\tau}^{P}\right)\right\},$$

$$\boldsymbol{\Xi}_{C} = \overline{\boldsymbol{Y}}^{NNJ}\left\{\boldsymbol{\Psi}_{\Delta}\left(\overline{\boldsymbol{Y}}^{ni^{-1}}\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}} + \theta^{X}\sigma\boldsymbol{\Omega}^{\top}\boldsymbol{\alpha}\right) - \left[\boldsymbol{I} + \theta^{X}\sigma\boldsymbol{\Omega}\right]\boldsymbol{\alpha}\right\}. \tag{B.6}$$

Stacking the country-level equations together with U.S.-bond market clearing gives the fifth equation in the five-equation representation. The $\boldsymbol{\Xi}_1$-$\boldsymbol{\Xi}_5$ coefficients in the fifth equation of the Global New Keynesian Representation, thus capture, how the balance of payments react to goods-specific terms of trade and to the balance sheet effect since $\overline{NX}_n = (1-\beta)\bar{V}_n^{US}$.

For illustration, in the $N = 2$ and $J = 1$ flexible-price version of our model, the fifth equation of the model—balance of payments, can be expressed in the following way with *symmetric* parameters:[36]

$$\begin{aligned}\beta\hat{V}_t = \hat{V}_{t-1} &+ \underbrace{\frac{\mathcal{A}}{1+\Omega}}_{\Xi_2}\left(\hat{C}_{H,t} - \hat{C}_{F,t}\right) + \underbrace{\frac{\mathcal{A}(\theta-1)}{1-\Omega}}_{\Xi_3}\left(\hat{p}_{H,t} - \hat{p}_{F,t}\right) \\ &+ \underbrace{\frac{\mathcal{A}(1+\Omega-2\theta)}{(1-\Omega)(1+\Omega)}}_{\Xi_4}\hat{\mathcal{E}}_t - \underbrace{\frac{\mathcal{A}\theta}{(1-\Omega)(1+\Omega)}}_{\Xi_5}\hat{\tau}_t\end{aligned} \tag{B.7}$$

where $\mathcal{A} \equiv \gamma + (1-\gamma)\Omega > 0$.

This expression demonstrates that elasticity of substitution, θ, determines the sign of Ξ_4, which in turn determines whether the Marshall-Lerner condition holds and a depreciation improves the trade balance. For intuition let us consider $\theta \to 0$: when goods are impossible to substitute, an exchange rate depreciation means imports (exports) become more expensive in domestic currency (less valuable in foreign currency), while export revenues in domestic currency (the import bill in foreign currency) remain the same. Via this mechanism depreciation can worsen the trade balance and increase the net debt of the home country, which corresponds to the case when $\Xi_4 > 0$ as $\theta \to 0$. A rise in the domestic-currency value of the import bill, with little offsetting quantity adjustment (decline in import quantity), can worsen the trade balance. Put differently, if a tariff mechanically pushes the external balance toward surplus, equilibrium may require a depreciation to offset that surplus.

[36]Under symmetry, we have symmetric coefficients inside the $\boldsymbol{\Xi}$ vectors. To demonstrate this with an example consider $\boldsymbol{\Xi}_2$, which is an $N \times 1$ vector. Under symmetry, we have $\boldsymbol{\Xi}_2 = [\Xi_{21}\Xi_{22}]'$, where $\Xi_{21} = -\Xi_{22} = \Xi_2$.

C Appendix for Section 3

C.1 Symmetry Case

Under incomplete markets,

$$\hat{\mathcal{E}}_t - (\hat{C}_{H,t} - \hat{C}_{F,t}) = \hat{w}_t.$$

Combining the Euler equations with UIP gives the Backus–Smith martingale condition

$$E_t \hat{w}_{t+1} = \hat{w}_t.$$

Under symmetry, $y_H = y_F = (1 - \Omega^2)^{-1}$ and

$$\begin{aligned}
\mathcal{A} &\equiv \gamma + (1-\gamma)\Omega, \\
\Xi_2 &= \frac{\mathcal{A}}{1+\Omega}, \\
\Xi_3 &= \frac{1}{1-\Omega}\left[\mathcal{A}(\theta^C - 1) + \frac{2\Omega}{1+\Omega}(\theta^X - \theta^C)\right], \\
\Xi_4 &= -(1-\gamma) - \frac{\theta^X - 1}{1-\Omega} - \frac{2\gamma\theta^C - \theta^X}{1+\Omega}, \\
\Xi_5 &= -\frac{L_\tau^C \, \gamma \, \theta^C}{1+\Omega} - \frac{L_\tau^P \, \Omega \, \theta^X}{1-\Omega^2}.
\end{aligned}$$

Solving the full open-economy block by MUC yields

$$\hat{w}_t = \frac{\beta - 1}{(\beta\rho_\tau - 1)\,\mathcal{D}} \mathcal{N} \, \varepsilon_t^\tau + \frac{(\beta-1)\rho_\tau}{(\beta\rho_\tau - 1)\,\mathcal{D}} \mathcal{N} \, \tau_{t-1} + \frac{(\beta-1)(1+\Omega)}{\mathcal{D}} \hat{V}_{H,t-1}, \qquad \text{(C.8)}$$

where

$$\mathcal{N} = L_\tau^C \gamma (1+\Omega)(\Xi_2 + \Xi_3 + \Xi_4) - L_\tau^P \Omega \Big[2\gamma(\Xi_2 + \Xi_3 + \Xi_4) - (\Xi_2 + \Xi_4)\Big] - (1+\Omega)\Xi_5,$$

$$\mathcal{D} = 2\Omega\gamma(\Xi_2 + \Xi_3 + \Xi_4) - 2\Omega\Xi_2 - \Omega\Xi_4 - 2\gamma(\Xi_2 + \Xi_3 + \Xi_4) + \Xi_4.$$

Hence, for every $k \geq 0$,

$$\frac{\partial E_t \hat{w}_{t+k}}{\partial \varepsilon_t^\tau} = \frac{1-\beta}{1-\beta\rho_\tau} \frac{\mathcal{N}}{\mathcal{D}}. \qquad \text{(C.9)}$$

Therefore

$$\Xi_2 + \Xi_3 + \Xi_4 = -\frac{\mathcal{A}(\theta^C - 1)}{1+\Omega},$$

and the wedge simplifies to $\forall k \geq 0$:

$$\frac{\partial E_t \hat{w}_{t+k}}{\partial \varepsilon_t^\tau} = \frac{1-\beta}{1-\beta\rho_\tau} \times \frac{(1+\Omega)\, L_\tau^C\, \gamma\big(\theta^C - \mathcal{A}(\theta^C - 1)\big) + L_\tau^P\, \Omega\big[\frac{2\mathcal{A}}{1-\Omega} + \theta^X - 2\gamma\mathcal{A} - 2\gamma(1-\gamma)(1-\Omega)\theta^C\big]}{\mathcal{A}(1-2\gamma)(1-\Omega) - 2\Omega\,\theta^X - 2\gamma(1-\gamma)(1-\Omega)^2\theta^C}$$

Setting $L_\tau^P = L_\tau^C = 1$ and $\theta^X = \theta^C = \theta$ gives (14) in the text.

Conditioning on a given constant wedge, the reduced static block implies

$$\hat{C}_{H,t} = -\frac{1}{1+\Omega}\Bigg\{\Big[L_\tau^C\, \gamma(1+\Omega) + L_\tau^P\, \frac{\Omega\big(1-\gamma(1-\Omega)\big)}{1-\Omega}\Big]\hat{\tau}_t + \big(\Omega(1-\gamma)+\gamma\big)\hat{w}_t\Bigg\}, \quad \text{(C.10)}$$

$$\hat{C}_{F,t} = \frac{1}{1+\Omega}\Bigg\{-L_\tau^P\, \frac{\Omega\big(\Omega(1-\gamma)+\gamma\big)}{1-\Omega}\,\hat{\tau}_t + \big(\Omega(1-\gamma)+\gamma\big)\hat{w}_t\Bigg\}, \quad \text{(C.11)}$$

$$\hat{\mathcal{E}}_t = \frac{1}{1+\Omega}\Bigg\{-\Big[L_\tau^C\, \gamma(1+\Omega) + L_\tau^P\, \Omega(1-2\gamma)\Big]\hat{\tau}_t + (1-\Omega)(1-2\gamma)\hat{w}_t\Bigg\}. \quad \text{(C.12)}$$

Setting $L_\tau^P = L_\tau^C = 1$ gives the expressions reported in Section 3.

C.2 With a Second Foreign Good

We add a second foreign good whose output is fixed and which enters production but not final consumption.

C.2.1 Scalar MUC

Notation. Here $C_1, \ldots, C_{18}$ are the scalar MUC coefficients from conjecturing linear rules in the two states $(\hat{V}_{H,t-1}, \hat{\tau}_t)$. Coefficients are numbered in pairs: the first coefficient multiplies $\hat{V}_{H,t-1}$, and the second multiplies $\hat{\tau}_t$. For the wedge calculation only the following decision rules matter:[37]

$$\hat{C}_{H,t} = C_1\hat{V}_{H,t-1} + C_2\hat{\tau}_t, \qquad \hat{C}_{F,t} = C_3\hat{V}_{H,t-1} + C_4\hat{\tau}_t,$$

$$\hat{\mathcal{E}}_t = C_{11}\hat{V}_{H,t-1} + C_{12}\hat{\tau}_t, \qquad \hat{V}_{H,t} = C_{17}\hat{V}_{H,t-1} + C_{18}\hat{\tau}_t.$$

Since

$$\hat{w}_t = \hat{\mathcal{E}}_t - (\hat{C}_{H,t} - \hat{C}_{F,t}),$$

[37] The missing intermediate coefficients are the analogous price and interest-rate coefficients, solved jointly in the scalar MUC system but not needed explicitly for the wedge expression.

the martingale property implies

$$\frac{\partial E_t \hat{w}_{t+k}}{\partial \varepsilon_t^\tau} = C_{12} - (C_2 - C_4) = -\frac{\left[(C_1 - C_3) - C_{11}\right] C_{18}}{1-\rho_\tau}, \qquad \forall\ k \geq 0.$$

Solving for the coefficients yields

$$\frac{\partial E_t \hat{w}_{t+k}}{\partial \varepsilon_t^\tau} = -\frac{1-\beta}{1-\beta\rho_\tau}\, \frac{(\Omega+\Omega_{H,T})\Big[\Omega\big(\Omega_{H,T}y(2-\theta^X)-\beta\big)+\alpha_F(A_2-\beta)\Big]-A_1\,A_3}{A_1\big(1-y\mathcal{D}\,\alpha_H\big)-\alpha_H\Big[\Omega\,\Omega_{H,T}y(2-\theta^X)+\alpha_F A_2\Big]},$$

where

$$\mathcal{D} \equiv \frac{1}{1+\Omega}, \qquad A_1 \equiv \Omega\Omega_{H,T}+\Omega\alpha_F+\Omega\alpha_H+\Omega_{H,T}\alpha_F+\alpha_F\alpha_H > 0,$$

$$A_2 \equiv y\Big(1+\Omega_{H,T}+\mathcal{D}\big[-\alpha_H-\theta^X(2\Omega+\Omega_{H,T})\big]\Big), \quad A_3 \equiv -y^2\mathcal{D}\,\theta^X L_\tau^P\big(\Omega\alpha_F+\Omega_{H,T}\big) < 0.$$

D Appendix for Section 4

Notation bridge to Section 4. This appendix solves the sticky-price block first in terms of the MUC coefficients $\{\boldsymbol{C}_1,\ldots,\boldsymbol{C}_{12}\}$ as it is easier to track numbered subscripts in algebra work. For compactness within this appendix, we redefine $\boldsymbol{\mu}_2$ to denote the full coefficient on $\Delta\hat{\boldsymbol{C}}_t$ in the real-marginal-cost equation: $\boldsymbol{\mu}_2 \equiv \sigma\Big(\boldsymbol{\alpha}+(\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^C+\boldsymbol{L}_{\mathcal{E}}^P)m\boldsymbol{Z}\Big), \qquad m \equiv \big(1-\boldsymbol{Z}\boldsymbol{L}_{\mathcal{E}}^C\big)^{-1}$. The coefficient on $\Delta\hat{\tau}_t$ is denoted by $\boldsymbol{\mu}_3 \equiv \boldsymbol{\alpha}\boldsymbol{L}_\tau^C+\boldsymbol{L}_\tau^P+(\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^C+\boldsymbol{L}_{\mathcal{E}}^P)m\boldsymbol{Z}\boldsymbol{L}_\tau^C = \sigma^{-1}\boldsymbol{\mu}_2\boldsymbol{L}_\tau^C+\boldsymbol{L}_\tau^P$. We label

$$\boldsymbol{C}_9 \equiv \boldsymbol{\Psi}^{\text{NKOE}}, \qquad \boldsymbol{C}_5 \equiv \boldsymbol{p}_\mu, \qquad \boldsymbol{C}_1 \equiv \boldsymbol{c}_\mu, \qquad \boldsymbol{C}_6 \equiv \boldsymbol{p}_w, \qquad \boldsymbol{C}_2 \equiv \boldsymbol{c}_w,$$

$$\boldsymbol{C}_7 \equiv \boldsymbol{p}_\tau, \qquad \boldsymbol{C}_3 \equiv \boldsymbol{c}_\tau, \qquad \boldsymbol{C}_{10} \equiv \boldsymbol{\mu}_w, \qquad \boldsymbol{C}_{11} \equiv \boldsymbol{\mu}_\tau,$$

with

$$\boldsymbol{C}_4 \equiv \boldsymbol{c}_{\tau,-1}, \qquad \boldsymbol{C}_8 \equiv \boldsymbol{p}_{\tau,-1}, \qquad \boldsymbol{C}_{12} \equiv \boldsymbol{\mu}_{\tau,-1}.$$

The matrix $\mathcal{K}(\rho,\boldsymbol{C}_9)$ is therefore the same object as $\mathcal{K}(\rho,\boldsymbol{\Psi}^{\text{NKOE}})$ in Section 4. In the decomposition formulas below we abbreviate it as $\boldsymbol{H}_\tau \equiv \mathcal{K}(\rho,\boldsymbol{\Psi}^{\text{NKOE}})$. With this bridge, the appendix formulas map directly into Propositions 2, 3, and 4.

D.1 Method of Undetermined Coefficients- New Keynesian Block

Postulate

$$\begin{aligned}
\Delta\hat{\boldsymbol{C}}_t &= \boldsymbol{C}_1\boldsymbol{\mu}_{t-1} + \boldsymbol{C}_2\Delta\hat{w}_t + \boldsymbol{C}_3\hat{\tau}_t + \boldsymbol{C}_4\hat{\tau}_{t-1},\\
\boldsymbol{\pi}_t^P &= \boldsymbol{C}_5\boldsymbol{\mu}_{t-1} + \boldsymbol{C}_6\Delta\hat{w}_t + \boldsymbol{C}_7\hat{\tau}_t + \boldsymbol{C}_8\hat{\tau}_{t-1},\\
\boldsymbol{\mu}_t &= \boldsymbol{C}_9\boldsymbol{\mu}_{t-1} + \boldsymbol{C}_{10}\Delta\hat{w}_t + \boldsymbol{C}_{11}\hat{\tau}_t + \boldsymbol{C}_{12}\hat{\tau}_{t-1}.
\end{aligned}$$

Using $E_t\hat{\tau}_{t+1} = \rho\hat{\tau}_t$ and $E_t\Delta\hat{w}_{t+1} = 0$, coefficient matching delivers the nonlinear system below.

D.2 System of 12 Equations and 12 Unknowns

Let $\boldsymbol{A}_2 \equiv (\boldsymbol{I} - \boldsymbol{L}_{\mathcal{E}}^C\boldsymbol{Z})\boldsymbol{\Phi}\boldsymbol{\Gamma}$. The coefficient restrictions are

$$\begin{aligned}
&0 = \sigma\boldsymbol{C}_1\boldsymbol{C}_9 - \boldsymbol{A}_2\boldsymbol{C}_5 + \boldsymbol{\Gamma}\boldsymbol{C}_5\boldsymbol{C}_9, && 0 = \boldsymbol{C}_5 - \boldsymbol{\Lambda}\boldsymbol{C}_9 - \beta\boldsymbol{C}_5\boldsymbol{C}_9,\\
&0 = \boldsymbol{C}_9 - \boldsymbol{I} - \boldsymbol{\mu}_1\boldsymbol{C}_5 - \boldsymbol{\mu}_2\boldsymbol{C}_1, && 0 = \sigma\boldsymbol{C}_1\boldsymbol{C}_{10} - \boldsymbol{A}_2\boldsymbol{C}_6 + \boldsymbol{\Gamma}\boldsymbol{C}_5\boldsymbol{C}_{10},\\
&0 = \boldsymbol{C}_6 - \boldsymbol{\Lambda}\boldsymbol{C}_{10} - \beta\boldsymbol{C}_5\boldsymbol{C}_{10}, && 0 = \boldsymbol{C}_{10} - \boldsymbol{\mu}_1\boldsymbol{C}_6 - \boldsymbol{\mu}_2\boldsymbol{C}_2 - \boldsymbol{\mu}_4,\\
&0 = \sigma(\boldsymbol{C}_1\boldsymbol{C}_{11} + \rho\boldsymbol{C}_3 + \boldsymbol{C}_4) - \boldsymbol{A}_2\boldsymbol{C}_7 + \boldsymbol{\Gamma}(\boldsymbol{C}_5\boldsymbol{C}_{11} + && \rho\,\boldsymbol{C}_7 + \boldsymbol{C}_8) - (1-\rho)\boldsymbol{L}_\tau^C,\\
&0 = \boldsymbol{C}_7 - \boldsymbol{\Lambda}\boldsymbol{C}_{11} - \beta(\boldsymbol{C}_5\boldsymbol{C}_{11} + \rho\boldsymbol{C}_7 + \boldsymbol{C}_8), && 0 = \boldsymbol{C}_{11} - \boldsymbol{\mu}_1\boldsymbol{C}_7 - \boldsymbol{\mu}_2\boldsymbol{C}_3 - \boldsymbol{\mu}_3,\\
&0 = \sigma\boldsymbol{C}_1\boldsymbol{C}_{12} - \boldsymbol{A}_2\boldsymbol{C}_8 + \boldsymbol{\Gamma}\boldsymbol{C}_5\boldsymbol{C}_{12}, && 0 = \boldsymbol{C}_8 - \boldsymbol{\Lambda}\boldsymbol{C}_{12} - \beta\boldsymbol{C}_5\boldsymbol{C}_{12},\\
&0 = \boldsymbol{C}_{12} - \boldsymbol{\mu}_1\boldsymbol{C}_8 - \boldsymbol{\mu}_2\boldsymbol{C}_4 + \boldsymbol{\mu}_3.
\end{aligned}$$

D.3 Defining Branches

For the NKOE propagation matrix it is enough to isolate the $\boldsymbol{\mu}_{t-1}$-block:

$$\sigma\boldsymbol{C}_1\boldsymbol{C}_9 = \boldsymbol{A}_2\boldsymbol{C}_5 - \boldsymbol{\Gamma}\boldsymbol{C}_5\boldsymbol{C}_9, \tag{D.13}$$

$$\boldsymbol{C}_5 = \boldsymbol{\Lambda}\boldsymbol{C}_9 + \beta\boldsymbol{C}_5\boldsymbol{C}_9, \tag{D.14}$$

$$\boldsymbol{C}_9 = \boldsymbol{I} + \boldsymbol{\mu}_1\boldsymbol{C}_5 + \boldsymbol{\mu}_2\boldsymbol{C}_1. \tag{D.15}$$

From (D.14),

$$\boldsymbol{C}_5 = \boldsymbol{\Lambda}\boldsymbol{C}_9(\boldsymbol{I} - \beta\boldsymbol{C}_9)^{-1}.$$

Substituting into (D.13)–(D.15), and using

$$\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2 \sigma^{-1} \boldsymbol{\Gamma} = \boldsymbol{\Omega} - \boldsymbol{I}, \qquad \boldsymbol{\mu}_2 \sigma^{-1} \boldsymbol{A}_2 = (\boldsymbol{\alpha} + \boldsymbol{L}_{\mathcal{E}}^{P} \boldsymbol{Z}) \boldsymbol{\Phi} \boldsymbol{\Gamma},$$

gives

$$\Big[(\boldsymbol{C}_9 - \boldsymbol{I})(\beta \boldsymbol{C}_9 - \boldsymbol{I}) - (\boldsymbol{I} - \boldsymbol{\Omega}) \boldsymbol{\Lambda} \boldsymbol{C}_9 + (\boldsymbol{\alpha} + \boldsymbol{L}_{\mathcal{E}}^{P} \boldsymbol{Z}) \boldsymbol{\Phi} \boldsymbol{\Gamma} \boldsymbol{\Lambda} \Big] \boldsymbol{C}_9 = \boldsymbol{0}. \tag{D.16}$$

This is $\mathcal{K}(0, \boldsymbol{C}_9)\boldsymbol{C}_9 = \boldsymbol{0}$ in the notation of the main text.

D.4 Branch 1: $\boldsymbol{C}_9 = \boldsymbol{0}$

If $\boldsymbol{C}_9 = \boldsymbol{0}$, then $\boldsymbol{C}_5 = \boldsymbol{0}$ and (D.15) reduces to

$$\boldsymbol{\mu}_2 \boldsymbol{C}_1 = -\boldsymbol{I}_{NJ}. \tag{D.17}$$

Hence the zero-propagation candidate exists only if $\boldsymbol{\mu}_2$ admits a right inverse. Since $\boldsymbol{\mu}_2 \in \mathbb{R}^{NJ \times N}$, for $J > 1$

$$\operatorname{rank}(\boldsymbol{\mu}_2 \boldsymbol{C}_1) \le \operatorname{rank}(\boldsymbol{\mu}_2) \le N < NJ,$$

so (D.17) is impossible. Therefore,

$$\boxed{J > 1 \quad \Longrightarrow \quad \boldsymbol{\Psi}^{\text{NKOE}} \neq \boldsymbol{0}.}$$

When $J = 1$, $\boldsymbol{\mu}_2 \in \mathbb{R}^{N \times N}$. Under the maintained regularity condition that $\boldsymbol{\mu}_2$ is nonsingular,

$$\boldsymbol{C}_1 = -\boldsymbol{\mu}_2^{-1}, \qquad \boldsymbol{C}_5 = \boldsymbol{0}, \qquad \boldsymbol{C}_9 = \boldsymbol{0}.$$

This is a stable solution, since all eigenvalues of $\boldsymbol{C}_9$ are zero. Because the linearized model is Blanchard–Kahn determinate, the stable equilibrium is unique. Hence

$$\boxed{J = 1 \quad \Longrightarrow \quad \boldsymbol{\Psi}^{\text{NKOE}} = \boldsymbol{0}.}$$

There may still be other formal roots of $\mathcal{K}(0, \boldsymbol{X})\boldsymbol{X} = \boldsymbol{0}$, but under determinacy they are not admissible equilibrium solutions.

D.5 Solving the rest of the system:

For $J > 1$, equilibrium must lie on the propagating branch $\boldsymbol{C}_9 = \boldsymbol{\Psi}^{\text{NKOE}} \neq 0$.

D.5.1 Branch 2: $\boldsymbol{C}_9 \neq \boldsymbol{0}$

We begin by guessing and verifying $\boldsymbol{C}_{12} = \boldsymbol{C}_8 = \boldsymbol{0}$. Then, assuming $\boldsymbol{\mu}_2$ has left inverse $\boldsymbol{\mu}_2^\ell$ and $\boldsymbol{\Lambda}$ is nonsingular, coefficient matching yields

$$\boldsymbol{C}_4 = \boldsymbol{\mu}_2^\ell \boldsymbol{\mu}_3, \qquad \boldsymbol{C}_5 = \boldsymbol{\Lambda}\boldsymbol{C}_9(\boldsymbol{I} - \beta\boldsymbol{C}_9)^{-1},$$

$$\mathcal{K}(\rho, \boldsymbol{C}_9) \equiv (1-\beta\rho)\Big[((1-\rho)\boldsymbol{I} - \boldsymbol{C}_9)(\boldsymbol{I} - \beta\boldsymbol{C}_9) - (\boldsymbol{I} - \boldsymbol{\Omega})\boldsymbol{\Lambda}\boldsymbol{C}_9\Big] + \Big[(\boldsymbol{\alpha} + \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{Z})\boldsymbol{\Phi}\boldsymbol{\Gamma} - \rho(\boldsymbol{I} - \boldsymbol{\Omega})\Big]\boldsymbol{\Lambda},$$

$$\boldsymbol{C}_7 = (1-\rho)\boldsymbol{\Lambda}\,\mathcal{K}(\rho, \boldsymbol{C}_9)^{-1}\boldsymbol{L}_\tau^P, \qquad \boldsymbol{C}_{11} = (1-\beta\rho)(\boldsymbol{I} - \beta\boldsymbol{C}_9)\boldsymbol{\Lambda}^{-1}\boldsymbol{C}_7,$$

$$\boldsymbol{C}_3 = \boldsymbol{\mu}_2^\ell \left\{(1-\rho)\Big[(1-\beta\rho)(\boldsymbol{I} - \beta\boldsymbol{C}_9) - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\Big]\mathcal{K}(\rho, \boldsymbol{C}_9)^{-1}\boldsymbol{L}_\tau^P - \boldsymbol{\mu}_3\right\},$$

$$\boldsymbol{C}_1 = \boldsymbol{\mu}_2^\ell\Big(\boldsymbol{C}_9 - \boldsymbol{I} - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\boldsymbol{C}_9(\boldsymbol{I} - \beta\boldsymbol{C}_9)^{-1}\Big).$$

For the $\Delta\hat{w}_t$-block define

$$\boldsymbol{H}_w(\boldsymbol{C}_9) \equiv \Big[\sigma\boldsymbol{\mu}_2^\ell\big((\boldsymbol{C}_9 - \boldsymbol{I})(\boldsymbol{I} - \beta\boldsymbol{C}_9) - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\boldsymbol{C}_9\big) - \boldsymbol{A}_2\boldsymbol{\Lambda} + \boldsymbol{\Gamma}\boldsymbol{\Lambda}\boldsymbol{C}_9\Big](\boldsymbol{I} - \beta\boldsymbol{C}_9 - \boldsymbol{\mu}_1\boldsymbol{\Lambda})^{-1}.$$

Then

$$\boldsymbol{C}_2 = -(\boldsymbol{H}_w(\boldsymbol{C}_9)\boldsymbol{\mu}_2)^{-1}\boldsymbol{H}_w(\boldsymbol{C}_9)\boldsymbol{\mu}_4,$$

$$\boldsymbol{C}_{10} = (\boldsymbol{I} - \beta\boldsymbol{C}_9)(\boldsymbol{I} - \beta\boldsymbol{C}_9 - \boldsymbol{\mu}_1\boldsymbol{\Lambda})^{-1}\big(\boldsymbol{\mu}_2\boldsymbol{C}_2 + \boldsymbol{\mu}_4\big), \quad \boldsymbol{C}_6 = \boldsymbol{\Lambda}(\boldsymbol{I} - \beta\boldsymbol{C}_9 - \boldsymbol{\mu}_1\boldsymbol{\Lambda})^{-1}\big(\boldsymbol{\mu}_2\boldsymbol{C}_2 + \boldsymbol{\mu}_4\big).$$

These are the coefficient formulas used in Propositions 2 and 4 once $\boldsymbol{C}_9 = \boldsymbol{\Psi}^{\text{NKOE}}$.

D.6 Decomposing the Impact on Inflation and Consumption

Since all endogenous variables are linear functions of $\hat{\tau}_t$ and $\Delta\hat{w}_t$, on impact we have:

$$\frac{\partial\hat{\boldsymbol{C}}_t}{\partial\hat{\tau}_t} = \boldsymbol{c}_\tau + \boldsymbol{c}_w w_\tau, \qquad \frac{\partial\boldsymbol{\pi}_t^P}{\partial\hat{\tau}_t} = \boldsymbol{p}_\tau + \boldsymbol{p}_w w_\tau, \qquad w_\tau \equiv \frac{\partial\Delta\hat{w}_t}{\partial\hat{\tau}_t}.$$

Let $\boldsymbol{\mu}_2^\ell \equiv (\boldsymbol{\mu}_2^\top\boldsymbol{\mu}_2)^{-1}\boldsymbol{\mu}_2^\top$. Then

$$\boldsymbol{p}_\mu = \boldsymbol{\Lambda}\boldsymbol{\Psi}^{\text{NKOE}}(\boldsymbol{I} - \beta\boldsymbol{\Psi}^{\text{NKOE}})^{-1}, \quad \boldsymbol{c}_\mu = \boldsymbol{\mu}_2^\ell\Big[\boldsymbol{\Psi}^{\text{NKOE}} - \boldsymbol{I} - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\boldsymbol{\Psi}^{\text{NKOE}}(\boldsymbol{I} - \beta\boldsymbol{\Psi}^{\text{NKOE}})^{-1}\Big],$$

$$\boldsymbol{c}_{\tau,-1} = \boldsymbol{\mu}_2^\ell\boldsymbol{\mu}_3, \qquad \boldsymbol{p}_{\tau,-1} = \boldsymbol{0}, \qquad \boldsymbol{\mu}_{\tau,-1} = \boldsymbol{0}.$$

Define

$$\boldsymbol{H}_\tau \equiv (1-\beta\rho)\big((1-\rho)\boldsymbol{I} - \boldsymbol{\Psi}^{\text{NKOE}}\big)(\boldsymbol{I} - \beta\boldsymbol{\Psi}^{\text{NKOE}})$$

$$+ (1-\beta\rho)(\boldsymbol{\Omega}-\boldsymbol{I})\boldsymbol{\Lambda}\boldsymbol{\Psi}^{\text{NKOE}} + \Big(\rho(\boldsymbol{\Omega}-\boldsymbol{I}) + (\boldsymbol{\alpha}+\boldsymbol{L}_{\mathcal{E}}^{P}\boldsymbol{Z})\boldsymbol{\Phi}\boldsymbol{\Gamma}\Big)\boldsymbol{\Lambda},$$

and

$$\begin{aligned}\boldsymbol{H}_w \equiv \Big[&\sigma\boldsymbol{\mu}_2^{\ell}\big((\boldsymbol{\Psi}^{\text{NKOE}}-\boldsymbol{I})(\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}}) - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\boldsymbol{\Psi}^{\text{NKOE}}\big) \\ &- \boldsymbol{A}_2\boldsymbol{\Lambda} + \boldsymbol{\Gamma}\boldsymbol{\Lambda}\boldsymbol{\Psi}^{\text{NKOE}}\Big](\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}}-\boldsymbol{\mu}_1\boldsymbol{\Lambda})^{-1}.\end{aligned}$$

Then

$$\boldsymbol{p}_\tau = (1-\rho)\boldsymbol{\Lambda}\boldsymbol{H}_\tau^{-1}\boldsymbol{L}_\tau^{P}, \quad \boldsymbol{c}_\tau = \boldsymbol{\mu}_2^{\ell}\left\{(1-\rho)\Big[(1-\beta\rho)(\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}}) - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\Big]\boldsymbol{H}_\tau^{-1}\boldsymbol{L}_\tau^{P} - \boldsymbol{\mu}_3\right\},$$

$$\boldsymbol{c}_w = -(\boldsymbol{H}_w\boldsymbol{\mu}_2)^{-1}\boldsymbol{H}_w\boldsymbol{\mu}_4, \qquad \boldsymbol{p}_w = \boldsymbol{\Lambda}(\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}}-\boldsymbol{\mu}_1\boldsymbol{\Lambda})^{-1}(\boldsymbol{\mu}_2\boldsymbol{c}_w+\boldsymbol{\mu}_4),$$

$$\boldsymbol{\mu}_w = (\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}})\boldsymbol{\Lambda}^{-1}\boldsymbol{p}_w, \qquad \boldsymbol{\mu}_\tau = (1-\beta\rho)(\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}})\boldsymbol{\Lambda}^{-1}\boldsymbol{p}_\tau.$$

D.6.1 Inflation

Let

$$\boldsymbol{\mathcal{Z}} \equiv \boldsymbol{L}_{\mathcal{E}}^{C}\, m\, \boldsymbol{Z}, \qquad \boldsymbol{R}_\tau \equiv \boldsymbol{\mu}_2^{\ell}\left\{(1-\rho)\Big[(1-\beta\rho)(\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}}) - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\Big]\boldsymbol{H}_\tau^{-1} - \boldsymbol{I}\right\},$$

so that $\boldsymbol{c}_\tau = -\boldsymbol{L}_\tau^{C} + \boldsymbol{R}_\tau\boldsymbol{L}_\tau^{P}$. Substituting the exchange-rate solution into CPI inflation and grouping terms yields

$$\begin{aligned}\frac{\partial \boldsymbol{\pi}_t^{C}}{\partial \hat{\tau}_t} = &\underbrace{\boldsymbol{\Gamma}\boldsymbol{L}_\tau^{P}}_{\text{Direct Effect on inputs}} + \underbrace{\left[\boldsymbol{\Gamma}\Big((1-\rho)\boldsymbol{\Lambda}\boldsymbol{H}_\tau^{-1} - \boldsymbol{I}\Big) + \boldsymbol{\mathcal{Z}}\Big(\boldsymbol{\Gamma}(1-\rho)\boldsymbol{\Lambda}\boldsymbol{H}_\tau^{-1} + \sigma\boldsymbol{R}_\tau\Big)\right]\boldsymbol{L}_\tau^{P}}_{\text{NKOE propagation}} \\ &+ \underbrace{\Big[\boldsymbol{I} + (1-\sigma)\boldsymbol{\mathcal{Z}}\Big]\boldsymbol{L}_\tau^{C}}_{\text{Contribution of tariffs on consumption}} + \underbrace{\Big[\boldsymbol{\Gamma}\boldsymbol{p}_w + \boldsymbol{\mathcal{Z}}\big(\sigma\boldsymbol{c}_w + \boldsymbol{\Gamma}\boldsymbol{p}_w\big) + \boldsymbol{L}_{\mathcal{E}}^{C}\, m\Big]w_\tau}_{\text{Contribution of the risk-sharing wedge}}.\end{aligned} \tag{D.18}$$

D.6.2 Consumption

Using the impact solution and $\boldsymbol{\mu}_3 = \boldsymbol{\mu}_2\boldsymbol{L}_\tau^{C} + \boldsymbol{L}_\tau^{P}$,

$$\frac{\partial \hat{\boldsymbol{C}}_t}{\partial \hat{\tau}_t} = \underbrace{-\boldsymbol{L}_\tau^{C}}_{L_\tau^{C}\text{ block}} + \underbrace{\boldsymbol{\mu}_2^{\ell}\left\{(1-\rho)\Big[(1-\beta\rho)(\boldsymbol{I}-\beta\boldsymbol{\Psi}^{\text{NKOE}}) - \boldsymbol{\mu}_1\boldsymbol{\Lambda}\Big]\boldsymbol{H}_\tau^{-1} - \boldsymbol{I}\right\}\boldsymbol{L}_\tau^{P}}_{L_\tau^{P}\text{ block}} + \underbrace{\boldsymbol{c}_w\, w_\tau}_{\hat{w}\text{ block}}. \tag{D.19}$$

D.7 Why input–output linkages further increase persistence

Proposition 5 is the extensive-margin result: when $J > 1$, real marginal-cost deviations can become inherited states. This subsection studies the intensive margin: conditional on that propagation channel being active, how does scaling up input–output linkages affect the speed at which inherited marginal-cost distortions decay?

Throughout this subsection, we take the passive-policy limit $\mathbf{\Phi} \to \mathbf{0}$ and compare networks along the scalar path $\mathbf{\Omega}(s) = s\,\bar{\mathbf{\Omega}},\ s \in [\underline{s}, \bar{s}]$, where the interval is chosen so that input shares remain admissible.

We use the following terminology. A root of $\mathbf{\Psi}(s)$ means an eigenvalue of $\mathbf{\Psi}(s)$. A *propagated root* is a nonzero root of $\mathbf{\Psi}(s)$. If $\mathbf{\Psi}(s)\boldsymbol{v} = \lambda\boldsymbol{v},\ \lambda \neq 0$, then a marginal-cost distortion proportional to $\boldsymbol{v}$ survives one period with decay factor λ. Zero roots instead correspond to directions eliminated in one step. Thus persistence is governed by the nonzero roots. An *eigenvalue branch* is a differentiable function such that $\lambda(s)$ is an eigenvalue of the relevant matrix for every s in the interval. When the eigenvalue is simple, this branch is locally well defined and differentiable.

Proposition 7 (Scalar scaling of input–output linkages)**.** *Let $\mathbf{\Psi}(s) \equiv \mathbf{\Psi}^{NKOE}(s)$ denote the stable solution to $\mathcal{K}(0, \mathbf{\Psi}(s))\mathbf{\Psi}(s) = \mathbf{0}$ under $\mathbf{\Omega}(s) = s\bar{\mathbf{\Omega}}$. Define $\boldsymbol{M}(s) \equiv (1+\beta)\boldsymbol{I} + (\boldsymbol{I} - s\bar{\mathbf{\Omega}})\mathbf{\Lambda}$. Let $\lambda(s) \in (0,1)$ be a real simple propagated-root branch of $\mathbf{\Psi}(s)$. Define $\zeta(s) \equiv \beta\lambda(s) + \lambda(s)^{-1}$. Then $\zeta(s)$ is an eigenvalue of $\boldsymbol{M}(s)$. Assume that $\zeta(s)$ is a simple eigenvalue branch of $\boldsymbol{M}(s)$. Let $\boldsymbol{x}(s)$ and $\boldsymbol{y}(s)$ denote right and left eigenvectors of $\boldsymbol{M}(s)$ associated with $\zeta(s)$, normalized by $\boldsymbol{y}(s)^\top\boldsymbol{x}(s) = 1$.*

If $\boldsymbol{y}(s)^\top\bar{\mathbf{\Omega}}\mathbf{\Lambda}\,\boldsymbol{x}(s) > 0$, then

$$\lambda'(s) = \frac{\lambda(s)^2}{1 - \beta\lambda(s)^2}\,\boldsymbol{y}(s)^\top\bar{\mathbf{\Omega}}\mathbf{\Lambda}\,\boldsymbol{x}(s) > 0.$$

Therefore scaling up input–output linkages raises the decay factor of this propagated marginal-cost mode.

Proof. Under $\mathbf{\Phi} = \mathbf{0}$,

$$\mathcal{K}(0, \boldsymbol{X}) = (\boldsymbol{I} - \boldsymbol{X})(\boldsymbol{I} - \beta\boldsymbol{X}) - (\boldsymbol{I} - \mathbf{\Omega})\mathbf{\Lambda}\boldsymbol{X}.$$

Along the path $\mathbf{\Omega}(s) = s\bar{\mathbf{\Omega}}$, this becomes $\mathcal{K}(0, \boldsymbol{X}) = \boldsymbol{I} - \boldsymbol{M}(s)\boldsymbol{X} + \beta\boldsymbol{X}^2$. Hence the equilibrium propagation matrix satisfies

$$\left[\boldsymbol{I} - \boldsymbol{M}(s)\mathbf{\Psi}(s) + \beta\mathbf{\Psi}(s)^2\right]\mathbf{\Psi}(s) = \mathbf{0}. \tag{D.20}$$

Take a propagated root $\lambda(s) \neq 0$ of $\boldsymbol{\Psi}(s)$, with right eigenvector $\boldsymbol{u}(s)$:

$$\boldsymbol{\Psi}(s)\boldsymbol{u}(s) = \lambda(s)\boldsymbol{u}(s).$$

This vector is a self-replicating pattern of real marginal-cost distortions. After one period, the same pattern remains, scaled by $\lambda(s)$. Thus a larger $\lambda(s) \in (0,1)$ means slower unwinding.

Apply (D.20) to $\boldsymbol{u}(s)$: $\big[\boldsymbol{I} - \boldsymbol{M}(s)\boldsymbol{\Psi}(s) + \beta\boldsymbol{\Psi}(s)^2\big]\boldsymbol{\Psi}(s)\boldsymbol{u}(s) = \mathbf{0}$. Using $\boldsymbol{\Psi}(s)\boldsymbol{u}(s) = \lambda(s)\boldsymbol{u}(s)$, this becomes

$$\lambda(s)\big[\boldsymbol{I} - \boldsymbol{M}(s)\boldsymbol{\Psi}(s) + \beta\boldsymbol{\Psi}(s)^2\big]\boldsymbol{u}(s) = \mathbf{0}.$$

Because $\lambda(s) \neq 0$, we may divide by this scalar. This is the key step: we do not invert or cancel the full matrix $\boldsymbol{\Psi}(s)$. We only divide by the nonzero scalar $\lambda(s)$. Hence

$$\big[\boldsymbol{I} - \boldsymbol{M}(s)\boldsymbol{\Psi}(s) + \beta\boldsymbol{\Psi}(s)^2\big]\boldsymbol{u}(s) = \mathbf{0}.$$

Using again $\boldsymbol{\Psi}(s)\boldsymbol{u}(s) = \lambda(s)\boldsymbol{u}(s)$ and $\boldsymbol{\Psi}(s)^2\boldsymbol{u}(s) = \lambda(s)^2\boldsymbol{u}(s)$, we obtain $\big[\boldsymbol{I} - \lambda(s)\boldsymbol{M}(s) + \beta\lambda(s)^2\boldsymbol{I}\big]\boldsymbol{u}(s) = \mathbf{0}$. Rearranging,

$$\boldsymbol{M}(s)\boldsymbol{u}(s) = \left(\beta\lambda(s) + \lambda(s)^{-1}\right)\boldsymbol{u}(s).$$

Thus $\zeta(s) \equiv \beta\lambda(s) + \lambda(s)^{-1}$ is an eigenvalue of $\boldsymbol{M}(s)$. Economically, $\zeta(s)$ is the effective damping term associated with the marginal-cost pattern $\boldsymbol{u}(s)$. A lower $\zeta(s)$ means weaker pullback toward steady state.

We now compute how $\zeta(s)$ moves with s. Let $\boldsymbol{x}(s)$ and $\boldsymbol{y}(s)$ be right and left eigenvectors of $\boldsymbol{M}(s)$ associated with $\zeta(s)$:

$$\boldsymbol{M}(s)\boldsymbol{x}(s) = \zeta(s)\boldsymbol{x}(s), \qquad \boldsymbol{y}(s)^\top\boldsymbol{M}(s) = \zeta(s)\boldsymbol{y}(s)^\top,$$

with normalization $\boldsymbol{y}(s)^\top\boldsymbol{x}(s) = 1$. To arrive at the standard simple-eigenvalue perturbation formula ($\zeta'(s) = \boldsymbol{y}(s)^\top\boldsymbol{M}'(s)\boldsymbol{x}(s)$), we begin by differentiating

$$\boldsymbol{M}(s)\boldsymbol{x}(s) = \zeta(s)\boldsymbol{x}(s)$$

with respect to s:

$$\boldsymbol{M}'(s)\boldsymbol{x}(s) + \boldsymbol{M}(s)\boldsymbol{x}'(s) = \zeta'(s)\boldsymbol{x}(s) + \zeta(s)\boldsymbol{x}'(s).$$

Premultiply by $\boldsymbol{y}(s)^\top$:

$$\boldsymbol{y}^\top \boldsymbol{M}'\boldsymbol{x} + \boldsymbol{y}^\top \boldsymbol{M}\boldsymbol{x}' = \zeta'\boldsymbol{y}^\top\boldsymbol{x} + \zeta\boldsymbol{y}^\top\boldsymbol{x}'.$$

Since $\boldsymbol{y}^\top\boldsymbol{M} = \zeta\boldsymbol{y}^\top$ and $\boldsymbol{y}^\top\boldsymbol{x} = 1$, the two terms involving $\boldsymbol{x}'$ cancel, leaving

$$\zeta'(s) = \boldsymbol{y}(s)^\top\boldsymbol{M}'(s)\boldsymbol{x}(s).$$

Because $\boldsymbol{M}(s) = (1+\beta)\boldsymbol{I} + (\boldsymbol{I} - s\bar{\boldsymbol{\Omega}})\boldsymbol{\Lambda}$, we have $\boldsymbol{M}'(s) = -\bar{\boldsymbol{\Omega}}\boldsymbol{\Lambda}$. Therefore

$$\zeta'(s) = -\boldsymbol{y}(s)^\top\bar{\boldsymbol{\Omega}}\boldsymbol{\Lambda}\,\boldsymbol{x}(s).$$

Thus, if $\boldsymbol{y}(s)^\top\bar{\boldsymbol{\Omega}}\boldsymbol{\Lambda}\,\boldsymbol{x}(s) > 0$, then increasing s lowers the effective damping term $\zeta(s)$.

Finally, differentiating the scalar relation $\zeta(s) = \beta\lambda(s) + \lambda(s)^{-1}$ yields:

$$\zeta'(s) = \left(\beta - \lambda(s)^{-2}\right)\lambda'(s) = -\frac{1-\beta\lambda(s)^2}{\lambda(s)^2}\lambda'(s).$$

Solving for $\lambda'(s)$,

$$\lambda'(s) = -\frac{\lambda(s)^2}{1-\beta\lambda(s)^2}\zeta'(s).$$

Substituting the expression for $\zeta'(s)$,

$$\lambda'(s) = \frac{\lambda(s)^2}{1-\beta\lambda(s)^2}\,\boldsymbol{y}(s)^\top\bar{\boldsymbol{\Omega}}\boldsymbol{\Lambda}\,\boldsymbol{x}(s).$$

Since $0 < \beta < 1$ and $0 < \lambda(s) < 1$,

$$\frac{\lambda(s)^2}{1-\beta\lambda(s)^2} > 0.$$

Hence the sign of $\lambda'(s)$ is the sign of $\boldsymbol{y}(s)^\top\bar{\boldsymbol{\Omega}}\boldsymbol{\Lambda}\,\boldsymbol{x}(s)$. Under the maintained sign condition, $\lambda'(s) > 0$. Thus scaling up intermediate-input use raises the fraction of this marginal-cost pattern that survives into the next period, which is precisely greater persistence. □

Applying Proposition 7 to a unique real simple dominant propagated-root branch $\lambda_*(s) \in (0,1)$ satisfying $\lambda_*(s) > \max\left\{|\lambda| : \lambda \in \sigma(\boldsymbol{\Psi}(s)),\ \lambda \neq 0,\ \lambda \neq \lambda_*(s)\right\}$ for every $s \in [\underline{s}, \bar{s}]$, with associated $\eta_*(s) = \beta\lambda_*(s) + \lambda_*(s)^{-1}$ a simple eigenvalue branch of $\boldsymbol{M}(s)$, proves Proposition 6.

D.8 Open-economy monetary-policy heterogeneity and persistence

This appendix demonstrates how monetary policy heterogeneity can change the persistence of real marginal cost deviations.

Proposition 8. *Consider $N = 2$ and $J > 1$. Suppose countries have identical target baskets. Let $\delta = \phi_\pi^H - \phi_\pi^F$ summarize policy heterogeneity; if $\delta = 0$, the policy-heterogeneity channel, $\boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{Z}\boldsymbol{\Phi}\boldsymbol{\Gamma}\boldsymbol{\Lambda}$, is eliminated. Let $\boldsymbol{\Psi}^{\mathrm{CE}}(\delta)$ and $\boldsymbol{\Psi}^{\mathrm{OE}}(\delta)$ solve the corresponding closed- and open-economy kernel conditions, with the open-economy kernel adding $\boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{Z}\boldsymbol{\Phi}(\delta)\boldsymbol{\Gamma}\boldsymbol{\Lambda}$. Let $\lambda_*^{\mathrm{CE}}(\delta)$ and $\lambda_*^{\mathrm{OE}}(\delta)$ be the local continuations of the largest real nonzero eigenvalue of the common $\delta = 0$ propagation matrix, and suppose this common eigenvalue $\lambda_* \in (0, 1)$ is simple. Let $\boldsymbol{A}_* \equiv \beta\lambda_*^2 \boldsymbol{I} - \lambda_* \boldsymbol{M} + \boldsymbol{I} + \phi_\pi \boldsymbol{\alpha}\boldsymbol{\Gamma}\boldsymbol{\Lambda}$, and let $\boldsymbol{v}_*$ and $\boldsymbol{u}_*$ satisfy $\boldsymbol{A}_*\boldsymbol{v}_* = \boldsymbol{0}$ and $\boldsymbol{u}_*^\top \boldsymbol{A}_* = \boldsymbol{0}^\top$. Then*

$$\partial_\delta[\lambda_*^{\mathrm{OE}}(\delta) - \lambda_*^{\mathrm{CE}}(\delta)]\big|_{\delta=0} = \frac{\boldsymbol{u}_*^\top \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda}\boldsymbol{v}_*}{\boldsymbol{u}_*^\top(\boldsymbol{M} - 2\beta\lambda_*\boldsymbol{I})\boldsymbol{v}_*}.$$

Hence, if $\boldsymbol{u}_*^\top \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda}\boldsymbol{v}_* \neq 0$, monetary-policy heterogeneity changes persistence; for small $\delta > 0$, it raises the largest real nonzero eigenvalue relative to the closed-economy policy block when the displayed ratio is positive, and lowers it when negative.

Proof. Fix $\boldsymbol{M} \equiv (1+\beta)\boldsymbol{I} + (\boldsymbol{I} - \boldsymbol{\Omega})\boldsymbol{\Lambda}$, $\boldsymbol{e}_H = (1, 0)^\top$, $\boldsymbol{e}_F = (0, 1)^\top$, $\boldsymbol{Z} = \boldsymbol{e}_H^\top - \boldsymbol{e}_F^\top$, and $\boldsymbol{\Phi}(\delta) = \phi_\pi \boldsymbol{I}_2 + \delta \boldsymbol{e}_H \boldsymbol{e}_H^\top = \left[\begin{smallmatrix} \phi_\pi + \delta & 0 \\ 0 & \phi_\pi \end{smallmatrix}\right]$. Identical target baskets imply $\boldsymbol{e}_H^\top \boldsymbol{\Gamma} = \boldsymbol{e}_F^\top \boldsymbol{\Gamma}$, so

$$\boldsymbol{Z}\boldsymbol{\Phi}(\delta)\boldsymbol{\Gamma}\boldsymbol{\Lambda} = (\boldsymbol{e}_H^\top - \boldsymbol{e}_F^\top)(\phi_\pi \boldsymbol{I}_2 + \delta \boldsymbol{e}_H \boldsymbol{e}_H^\top)\boldsymbol{\Gamma}\boldsymbol{\Lambda} = \delta \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda},$$

and therefore $\boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{Z}\boldsymbol{\Phi}(\delta)\boldsymbol{\Gamma}\boldsymbol{\Lambda} = \delta \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda}$, which is zero at $\delta = 0$.

Define

$$\boldsymbol{F}^{\mathrm{CE}}(\lambda, \delta) \equiv \beta\lambda^2 \boldsymbol{I} - \lambda \boldsymbol{M} + \boldsymbol{I} + \boldsymbol{\alpha}\boldsymbol{\Phi}(\delta)\boldsymbol{\Gamma}\boldsymbol{\Lambda}, \qquad \boldsymbol{F}^{\mathrm{OE}}(\lambda, \delta) \equiv \boldsymbol{F}^{\mathrm{CE}}(\lambda, \delta) + \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{Z}\boldsymbol{\Phi}(\delta)\boldsymbol{\Gamma}\boldsymbol{\Lambda}.$$

At $\delta = 0$, the two pencils coincide, and $\boldsymbol{F}^{\mathrm{CE}}(\lambda_*, 0) = \boldsymbol{F}^{\mathrm{OE}}(\lambda_*, 0) = \boldsymbol{A}_*$. For $s \in \{\mathrm{CE}, \mathrm{OE}\}$, the kernel condition for the local branch gives $\boldsymbol{F}^s(\lambda_*^s(\delta), \delta)\boldsymbol{v}^s(\delta) = \boldsymbol{0}$. Differentiating at $\delta = 0$ and premultiplying by $\boldsymbol{u}_*^\top$ gives, with the denominator nonzero by simplicity,

$$\left.\frac{d\lambda_*^s(\delta)}{d\delta}\right|_{\delta=0} = \frac{\boldsymbol{u}_*^\top \, \partial_\delta \boldsymbol{F}^s(\lambda, \delta)|_{(\lambda,\delta)=(\lambda_*,0)} \, \boldsymbol{v}_*}{\boldsymbol{u}_*^\top(\boldsymbol{M} - 2\beta\lambda_*\boldsymbol{I})\boldsymbol{v}_*}.$$

The required partial derivatives are

$$\partial_\delta \boldsymbol{F}^{\mathrm{CE}}(\lambda, \delta)\big|_{(\lambda,\delta)=(\lambda_*,0)} = \boldsymbol{\alpha}\boldsymbol{e}_H \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda}, \qquad \partial_\delta \boldsymbol{F}^{\mathrm{OE}}(\lambda, \delta)\big|_{(\lambda,\delta)=(\lambda_*,0)} = \boldsymbol{\alpha}\boldsymbol{e}_H \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda} + \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda}.$$

Subtracting the closed-economy derivative from the open-economy derivative yields

$$\frac{d}{d\delta}\left[\lambda_*^{\mathrm{OE}}(\delta) - \lambda_*^{\mathrm{CE}}(\delta)\right]\Big|_{\delta=0} = \frac{\boldsymbol{u}_*^\top \boldsymbol{L}_{\mathcal{E}}^P \boldsymbol{e}_H^\top \boldsymbol{\Gamma}\boldsymbol{\Lambda}\boldsymbol{v}_*}{\boldsymbol{u}_*^\top(\boldsymbol{M} - 2\beta\lambda_*\boldsymbol{I})\boldsymbol{v}_*}.$$

If the numerator is nonzero, the open-economy policy heterogeneity term changes the local response of the propagated eigenvalue to monetary-policy heterogeneity. Writing the displayed ratio as $\mathcal{R}$, $\lambda_*^{\mathrm{OE}}(\delta) - \lambda_*^{\mathrm{CE}}(\delta) = \delta\mathcal{R} + o(\delta)$. Since $\lambda_* \in (0,1)$ is the simple largest real nonzero eigenvalue at $\delta = 0$, continuity of the local branch implies that, for sufficiently small δ, this branch remains the relevant largest real nonzero eigenvalue. Therefore, for small $\delta > 0$, the policy heterogeneity term raises the largest real nonzero eigenvalue relative to the closed-economy policy block when $\mathcal{R} > 0$, and lowers it when $\mathcal{R} < 0$. This proves Proposition 8. □

D.9 Solving for the Risk-Sharing Wedge- Open Economy Block

Using

$$\hat{\mathcal{E}}_t = m\left[\boldsymbol{Z}\left(\sigma\hat{\boldsymbol{C}}_t + \boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_\tau^C\hat{\tau}_t\right) + \hat{w}_t\right], \qquad m \equiv \left(1 - \boldsymbol{Z}\boldsymbol{L}_{\mathcal{E}}^C\right)^{-1},$$

the balance-of-payments block can be written as

$$\beta\hat{V}_t = \hat{V}_{t-1} + \widehat{NX}_t + \boldsymbol{\Xi}_6\boldsymbol{\Phi}\boldsymbol{\Gamma}\boldsymbol{\pi}_t^P,$$

$$\widehat{NX}_t = \widehat{NX}_{t-1} + \tilde{\boldsymbol{\Xi}}_2\Delta\hat{\boldsymbol{C}}_t + \tilde{\boldsymbol{\Xi}}_3\boldsymbol{\pi}_t^P + \tilde{\Xi}_4\Delta\hat{w}_t + \tilde{\Xi}_5(\hat{\tau}_t - \hat{\tau}_{t-1}),$$

where

$$\tilde{\boldsymbol{\Xi}}_2 \equiv \boldsymbol{\Xi}_2 + m\sigma\Xi_4\boldsymbol{Z}, \qquad \tilde{\boldsymbol{\Xi}}_3 \equiv \boldsymbol{\Xi}_3 + m\Xi_4\boldsymbol{Z}\boldsymbol{\Gamma}, \qquad \tilde{\Xi}_4 \equiv m\Xi_4.$$

Combining this block with the NK solution above yields the wedge coefficient.

D.9.1 Solving for the Wedge

Let $\Delta\hat{w}_t = C_{26}\hat{\tau}_t$ on impact. In the nondegenerate martingale branch with $(\boldsymbol{C}_9 - \boldsymbol{I})$ invertible,

$$C_{26} = \frac{\tilde{\boldsymbol{\Xi}}_2(\boldsymbol{C}_3 + \boldsymbol{C}_4) + \tilde{\boldsymbol{\Xi}}_3(\boldsymbol{C}_7 + \boldsymbol{C}_8) - \left(\tilde{\boldsymbol{\Xi}}_2\boldsymbol{C}_1 + \tilde{\boldsymbol{\Xi}}_3\boldsymbol{C}_5\right)(\boldsymbol{C}_9 - \boldsymbol{I})^{-1}(\boldsymbol{C}_{11} + \boldsymbol{C}_{12})}{(1-\rho)\Big[\left(\tilde{\boldsymbol{\Xi}}_2\boldsymbol{C}_1 + \tilde{\boldsymbol{\Xi}}_3\boldsymbol{C}_5\right)(\boldsymbol{C}_9 - \boldsymbol{I})^{-1}\boldsymbol{C}_{10} - \tilde{\boldsymbol{\Xi}}_2\boldsymbol{C}_2 - \tilde{\boldsymbol{\Xi}}_3\boldsymbol{C}_6 - \tilde{\Xi}_4\Big]}. \tag{D.21}$$

Under the Branch-2 restriction $\boldsymbol{C}_8 = \boldsymbol{C}_{12} = \boldsymbol{0}$, this becomes ($\forall\ k \geq 0$)

$$\frac{\partial \hat{w}_{t+k}}{\partial \hat{\tau}_t} = C_{26} = \frac{\tilde{\boldsymbol{\Xi}}_2(\boldsymbol{C}_3 + \boldsymbol{C}_4) + \tilde{\boldsymbol{\Xi}}_3\boldsymbol{C}_7 - \big(\tilde{\boldsymbol{\Xi}}_2\boldsymbol{C}_1 + \tilde{\boldsymbol{\Xi}}_3\boldsymbol{C}_5\big)(\boldsymbol{C}_9 - \boldsymbol{I})^{-1}\boldsymbol{C}_{11}}{(1-\rho)\Big[\big(\tilde{\boldsymbol{\Xi}}_2\boldsymbol{C}_1 + \tilde{\boldsymbol{\Xi}}_3\boldsymbol{C}_5\big)(\boldsymbol{C}_9 - \boldsymbol{I})^{-1}\boldsymbol{C}_{10} - \tilde{\boldsymbol{\Xi}}_2\boldsymbol{C}_2 - \tilde{\boldsymbol{\Xi}}_3\boldsymbol{C}_6 - \tilde{\Xi}_4\Big]} \tag{D.22}$$

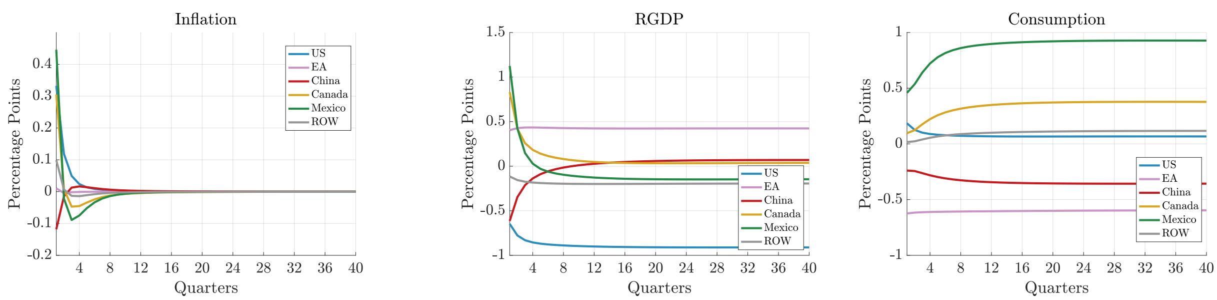
E Additional Results

E.1 Announced Liberation Day Tariffs

Figure E.1 reports the baseline simulation for the announced 2025 U.S. tariff package. On impact, U.S. real GDP falls by 0.65%, inflation rises by 0.33 percentage points, consumption rises by 0.18%, and the trade-weighted NEER appreciates by 11.31%. Cross-country spillovers are heterogeneous: China contracts, Mexico and Canada expand, and the euro area and the rest of the world move modestly.

Figure E.1. Impact of Liberation Day Tariffs

(a) **(b)** **(c)**



NOTE: Simulated responses to the announced 2025 U.S. tariff package under MIT shocks.

F Additional Tables and Figures

Table F.1. Descriptive Statistics for the U.S. (%)

Industry	Output Share	VA Share	Consumption Share	Output Home Share	Consumption Home Share	Intermediate Home Share
Agriculture	1.3	0.9	0.6	87.2	88.5	89.3
Energy	3.0	2.0	1.5	85.7	89.4	75.0
Mining	0.5	0.5	0.5	91.2	98.5	89.9
Food and Beverages	2.6	1.2	3.1	94.0	91.2	91.7
Basic Manufacturing	6.6	4.7	4.1	87.6	66.0	82.5
Advanced Manufacturing	6.2	5.1	8.2	81.7	67.0	66.9
Residential Services	6.4	6.1	7.7	99.9	99.9	99.5
Services	73.4	79.4	74.3	95.3	96.7	96.2

NOTES: 'Output Share' is the share of the sector in total U.S. output. 'VA Share' is the share of the sector in total U.S. GDP. 'Consumption Share' is calculated as the sector's weight in the household expenditure. 'Output Home Share' represents the share of the output of the sector sold domestically. 'Consumption Home Share' captures the share of domestic production in consumption and 'Intermediate Home Share' captures the share of intermediate goods supplied domestically.